Recurrent method with GNSS data for 3D displacement analysis and prediction time-variable dynamic model in Central Vietnam

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SUMMARY

Nowadays, with the application of GNSS technology, solving the displacement problem is not only detecting, but also monitoring and studying the displacement process according to variables. With the impact of climate change, natural disasters and the impact of rising sea levels, in some areas of Vietnam, deformation displacement phenomena such as subsidence and horizontal displacement have appeared. In the report based on the use of 3D data obtained from the results of monitoring network measurements using GNSS technology according to time cycles, the focus is on analyzing the 3D dynamic displacement model of a region in the Central region of Vietnam. The new point of the proposed solution is to build a recursive method combining Kanmal filter theory to calculate and determine displacement parameters in the dynamic model. The displacement analysis was tested in 3 measurement cycles and the forecast results were compared with the observation results of cycle 4. The research results can serve for basic investigation in localities with the possibility of subsidence or deformation due to geological faults or environmental impacts.

Keywords: GNSS application, Adjustment computation,National geodetic network, terrestrial measurement systems.

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1 INTRODUCTION

From the 1980s and 1990s, Professor Markuze YU.I [11] proposed and developed the theory of recurrent adjustment with outstanding advantages in filtering gross errors and adjusting large geodetic networks. With the application of GNSS technology, network construction achieves high accuracy in a short time and can be continuously observed, in a wide range and can determine the displacement vector in the dynamic model.

For local 3D analysis, it is more convenient to use geocentric coordinates, especially for vertical displacement analysis. Some publications have mentioned the application of Kalman filtering to apply in the field of deformation analysis [1],[3],[5].[14]. The adjustment of the spatial grid and displacement analysis in the topocentric coordinate system allows for a better assessment of horizontal or vertical displacement, thus determining the subsidence of the land area under study.

The paper presents a solution to develop the theory of recurrent adjustment in the analysis of dynamic model deformation displacement of GNSS monitoring network with 4 measurement cycles.

2. Methodology

2.1. Dynamic model of the deformation analysis with recurrent adjustment

The vector of coordinates of the periodic observation (k+1) is determined from the vector of coordinates of the periodic observation (k):

$$x_j^{(k+1)} - \Phi\left(t^{(k+1)}\right) \quad , x_j^{(k)} = \Phi\left(t^{(k)}\right) \qquad (1)$$

Implementing the Taylor formula of the function of the vector of coordinates of the periodic observation $x_j^{(k+1)}$ according to the time variable t, the dynamic model over time with coordinates, velocity and acceleration is represented by the following formula:

$$\Phi\left(t^{(k+1)}\right) = C\left(t^{(k)}\right) + \frac{\partial \Phi}{\partial \mathrm{t}}\left(t^{(k+1)} - t^{(k)}\right) + 0{,}5.\frac{\partial^2 \Phi}{\partial t^2}\left(\left(t^{(k+1)} - t^{(k)}\right)^2 \qquad (2)$$

$\frac{\partial \Phi}{\partial t} = v_{x_j}^{(k+1)}; \frac{\partial^2 \Phi}{\partial t^2} = a_{x_j}^{(k+1)}.$:(3)

v_{xj} : velocities of vector coordinates of point j

a_{xj}: accelerations of vector coordinates of point j

k=1, 2, . . . , i (i: measurement period number)

j=1, 2, . . . , k (k: number of points)

From formula (19), the following system of equations can be established:

$$x_j^{(k+1)} = x_j^{(k)} + (t_{k+1-}t_k)v_{x_j}^{(k+1)} + 0{,}5(t_{k+1-}t_k)^2 a_{x_j}^{(k+1)}$$

: $$v_{x_j}^{(k+1)} = v_{x_j}^{(k)} + (t_{k+1-}t_k)a_{x_j}^{(k)}$$

$$ax_{x_j}^{(k+1)} = a_{x_j}^{(k)}$$

(4)

Expression (4) can be rewritten in matrix form as follows:

$$\begin{pmatrix} x_j^{(k+1)} \\ v_{x_j}^{(k+1)} \\ a_{x_j}^{(k+1)} \end{pmatrix} = \begin{pmatrix} E & (t_{k+1-}t_k)\mathrm{E} & 0{,}5(t_{k+1-}t_k)^2 E \\ 0 & \mathrm{E} & \mathrm{E} \\ 0 & 0 & E \end{pmatrix} \begin{pmatrix} x_j^{(k)} \\ v_{x_j}^{(k)} \\ a_{x_j}^{(k)} \end{pmatrix} \quad (5)$$

E-The unite matrix

The Symbol:

$$y_j^{(k+1)} = \begin{pmatrix} x_j^{(k+1)} \\ v_{x_j}^{(k+1)} \\ a_{x_j}^{(k+1)} \end{pmatrix} \quad (6)$$

$$y_j^{(k)} = \begin{pmatrix} x_j^{(k)} \\ v_{x_j}^{(k)} \\ a_{x_j}^{(k)} \end{pmatrix} \quad (7)$$

$$y_j^{(k+1)} = G_j^{(k+1)} y_j^{(k)} \quad (8)$$

$$\bar{\mathrm{Y}}_{(\mathrm{k+1})} = \mathrm{G}_{(\mathrm{k+1})} \hat{\mathrm{Y}}_{(k+1)} \quad (9)$$

Here:

$$\bar{\mathrm{Y}}_{(k+1)} = \begin{pmatrix} y_1^{(k+1)} \\ y_2^{(k+1)} \\ \dots \\ y_n^{(k+1)} \end{pmatrix} \quad (10)$$

$$\bar{\mathrm{Y}}_{(k)} = \begin{pmatrix} y_1^{(k)} \\ y_2^{(k)} \\ \dots \\ y_n^{(k)} \end{pmatrix} \quad (11)$$

$$\mathrm{G}_{(\mathrm{k+1})} = \begin{pmatrix} \mathbf{G_1^{(k+1)}} & & \\ & \dots & \\ & & \mathbf{G_n^{(k+1)}} \end{pmatrix} \quad (12)$$

$$\bar{\mathrm{Y}}_{(k+1)} = \begin{pmatrix} y_1^{(k+1)} \\ y_2^{(k+1)} \\ \dots \\ y_n^{(k+1)} \end{pmatrix} \quad (13)$$

$$\bar{\mathrm{Y}}_{(k)} = \begin{pmatrix} y_1^{(k)} \\ y_2^{(k)} \\ \dots \\ y_n^{(k)} \end{pmatrix} \quad (14)$$

$$\mathrm{G}_{(\mathrm{k+1})} = \begin{pmatrix} \mathrm{G_1^{(k+1)}} & & \\ & \dots & \\ & & \mathrm{G_n^{(k+1)}} \end{pmatrix} \quad (15)$$

According to Kalman filter theory [1], [14], if using the calculation results at period tk, then according to formula (21), predict the current state vector by using the state vector information of the known motion parameters at period tk and the measurements at period t_{k+1}. collected at period t_{k+1}.. The matrix equation system of the motion model used to predict the motion parameters by Kalman filter technique in the grid can be represented as follows:

$$\bar{Y}_{(k+1)} = G_{(k+1)}\hat{Y}_k + S_{k+1,k} \tag{16}$$

$$C_{\bar{Y}(k+1)} = G_{k+1,k}\, C_Y G^T_{k+1,k}$$

(17)

$\hat{Y}^{(k)}$ is the vector of calculated values adjusted at time t_k.

$\bar{Y}^T_{(k+1}$ - vector of predicted values of coordinates, velocity and acceleration$\hat{Y}^T_k$ -vector of averaged values at time t_k.

At time t_{k+1}, we can use the geodetic grid adjustment results in the previous observation period (t_k) and establish a system of equations of correction numbers according to the least squares method with new measurement values. Thus, at time t_{k+1}, we can consider the geodetic grid points to have been virtually measured with measurement values and weights as the grid adjustment results in this period. Then we have the coefficient matrix of the system of equations of correction numbers in the following form:

$$\hat{A}_{k+1} = (E_{3x3} \quad O \quad O \ldots\ldots E_{3x3} \quad O \quad O) \tag{18}$$

$$V = \hat{A}_{k+1}\hat{Y}_{k+1} + L \tag{19}$$

$$\begin{pmatrix} V_{\bar{Y}(k+1)} \\ V_{L_{k+1}} \end{pmatrix} = \begin{pmatrix} E \\ A_{(k+1)} \end{pmatrix} \bar{Y}_{(k+1)} + \begin{pmatrix} O \\ L_{k+1} \end{pmatrix} \tag{20}$$

The weight matrix:

$$P = \begin{pmatrix} P_{\bar{Y}(k+1)} & O \\ O & P_{L_{k+1}} \end{pmatrix} \tag{21}$$

$$P_{\bar{Y}(k+1)} = Q_{\bar{Y}(k+1)}{}^{-1} \tag{22}$$

$$P_{L_{k+1}} = Q_{L_{k+1}}{}^{-1} \tag{23}$$

The normal system of equations has the form:

$$\left(\mathrm{P}_{\bar{Y}(k+1)} + A_{(k+1)}{}^{T}\, \mathrm{P}_{L_{k+1}}\, A_{(k+1)}\right)\Delta\hat{Y}_{k+1} + A_{(k+1)}{}^{T}\, \mathrm{P}_{L_{k+1}} L_{k+1} = O$$

(24)

By the recursive formula [11](.Markuze, YU.I, Hoang H)

$$\Delta\hat{Y}_{k+1} = -\left(\mathrm{P}_{\bar{Y}(k+1)} + A_{(k+1)}{}^{T}\, \mathrm{P}_{L_{k+1}}\, A_{(k+1)}\right)^{-1} A_{(k+1)}{}^{T}\, \mathrm{P}_{L_{k+1}} L_{k+1}$$

(25)

$$\left(\mathrm{P}_{\bar{Y}(k+1)} + A_{(k+1)}{}^{T}\, \mathrm{P}_{L_{k+1}}\, A_{(k+1)}\right)^{-1} = \mathrm{Q}_{\bar{Y}(k+1)} - \mathrm{Q}_{\bar{Y}(k+1)} A_{(k+1)} (\mathrm{Q}_{L_{k+1}} + \left(\mathrm{Q}_{L_{k+1}}{}^{-1} + A_{(k+1)} \mathrm{Q}_{\bar{Y}(k+1)} A_{(k+1)}{}^{T}\right)^{-1} A_{(k+1)}{}^{T} \mathrm{Q}_{\bar{Y}(k+1)}$$

(26)

$$\Delta\hat{Y}_{k+1} = -Z_{k+1}^{T}\, N_{k+1}^{-1} L_{k+1}$$

(27)

$$Q_{\hat{Y}_{k+1}} = \mathrm{Q}_{\bar{Y}(k+1)} - Z_{(k+1)}\, N_{k+1}^{-1}\, Z_{(k+1)}{}^{T}$$

(28)

Here:

$$Z_{(k+1)} = \mathrm{Q}_{\bar{Y}(k+1)} A_{(k+1)}{}^{T}$$

(29)

$$N_{(k+1)} = Q_{\bar{Y}(k+1)}{}^{-1} + Z_{(k+1)}{}^{T} A_{(k+1)}{}^{T}$$

(30)

$$\hat{Y}_{k+1} = \bar{Y}_{(k+1)} + \Delta\hat{Y}_{k+1}$$

(31)

2.2. GNSS network adjustment in the topocentric coordinate system

For convenience in analyzing elevation deformation, we perform grid adjustments in cycles within the topocentric coordinate system.Free adjustment of GNSS spatial grid in geocentric coordinate system (X,Y,Z) we will have a system of equations of corrections for n baselines has the following form:

$$V_{nx1} = A_{n\times k}\Delta x_{kx1} + L_{nx1} \quad (32)$$

Here A - the coefficient matrix

$\Delta \mathbf{x_{kx1}}$ is the vector of unknowns, V_{nx1} is vector of corrected numbers; k=3xt.

The weight matrix P has the form:

$$P = \begin{pmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & P_n \end{pmatrix} \quad (33)$$

$$P_i = Q_i^{-1} \quad (34)$$

Q_i - the cofactor matrix of the i -th baseline measurements

For each point of the GNSS grid, we have the following formula:

$$\begin{pmatrix} U_i \\ N_i \\ E_i \end{pmatrix} = \begin{pmatrix} -\sin B_0 \cos L_0 & \cos B_0 sin L_0 & \cos B_0 \\ -sin L_0 & cos\ L_0 & 0 \\ cos B_0 cos L_0 & \cos B_0\ sin & -\ sin B_0 \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} = \Lambda_i \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}$$
(35)

X_0, Y_0, Z_0 - The coordinates of the origin of the topocentric coordinate systemin the geocentric coordinate system.

U_i, N_i, E_i - The coordinates of the topocentric coordinate system (Figure 1)

Thus, if the unknowns are U, N, E, the system of equations (1) can be rewritten as follows:

$$V = A_z \Delta z + L \quad (36)$$

$$A_z = \Lambda\, A_z \quad (37)$$

$$\Lambda = \begin{pmatrix} \Lambda_1 & & & \\ & \Lambda_2 & & \\ & \cdots & & \\ & & & \Lambda_n \end{pmatrix} \quad (38)$$

The normal system of equations has the form:

$$R_z \Delta z + b_z = 0 \quad (39)$$

$$R_z = A_Z^T\, P\, A_z, b_z = A_Z^T\, PL \quad (40)$$

$$\Delta z = -R_z^{\sim} b_z \tag{41}$$

$R_z^{\sim}$ - is the general inverse matrix.

$$R_z^{\sim} = (R_z + CC^T)^{-1} - TT^T \tag{42}$$

$$T^T = B(C^T G)^{-1} \tag{43}$$

$$G^T = (G_1 \quad G_2 \quad \dots \quad G_n) \tag{44}$$

Where:

n is the number of measurements.

$$G_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{45}$$

$$C^T = (G_1 \quad \cdots \quad G_k \quad 0 \quad \dots \quad 0) \tag{46}$$

For an accurate assessment, the following quantities need to be calculated:

$$s_0 = \sqrt{\frac{V^T PV}{n-k+d}} \tag{47}$$

d is the number of defects of the network (d = 3).

The covariance matrix is :

$$K_z = s_0^2 Q_z \tag{48}$$

$Q_x = R_z^{\sim}$ - The cofactor matrix

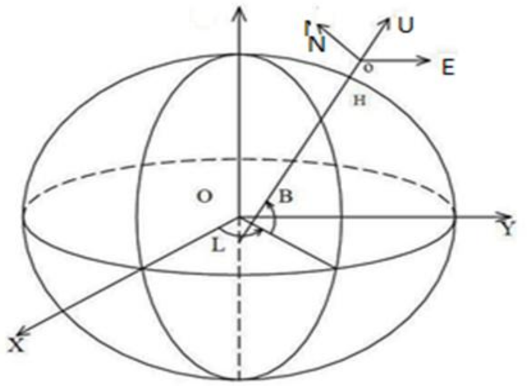

Figure 1. The topocentric coordinate system

2.3. Data processing procedures

Step 1: Data preprocessing of GNSS adjustment at each measurement cycle

- Input:

- Geodetic Datum (the local topocentric coordinate system)
- Priori coordinates of GNSS points
- GNSS observations.

- Output

- Estimated value of coordinates of GNSS points in the local topocentric coordinate system
- RMS error

Step 2: Calculate the parameters dU, dN, dE, v_U, v_N, v_E, a_U, a_N, a_E over the periods

Step 3: Displacement analysis based on statistical criteria

Step 3: the Prediction

3. Results

3.1. GNSS networks adjustment in the local topocentric coordinate system

To conduct experimental research and analyze the displacement according to the theory presented above, we have performed on GNSS measurement data in the central region of Vietnam (Thach Ban - Cat Tien, Lam Dong province). The monitoring network consists of 6 points (Figure 2). The construction of the network and the measurement was carried out by the Vietnam Institute of Geodesy and Cartography. The landmark is built according to the standard of mandatory centering landmarks placed on the bedrock. Repeat measurement for 4 cycles 2015, 2016, 2017 and 2018, time interval between cycles is one year.

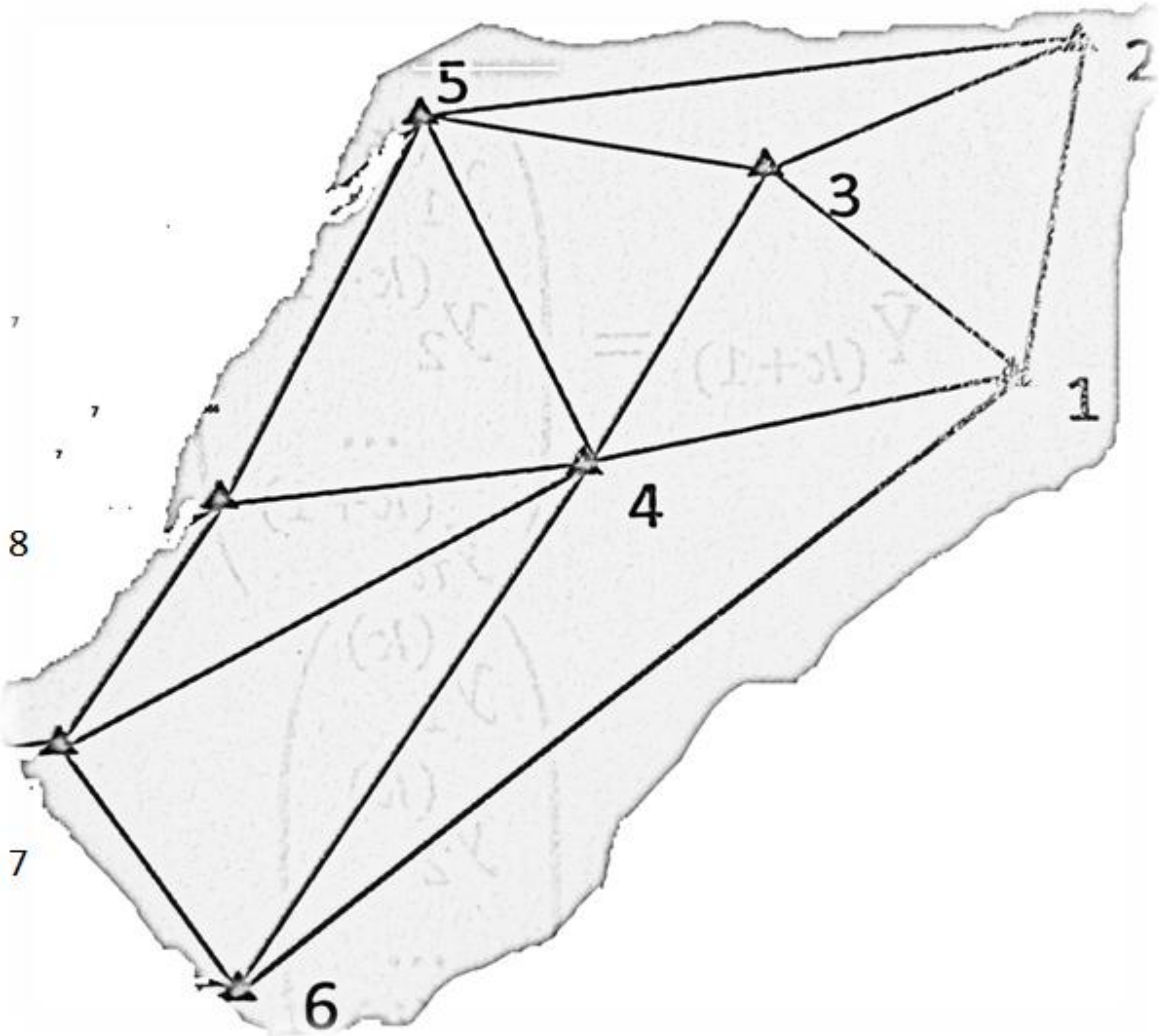

Figure 2. GNSS network Diagram

In the first step, applying the 3D free network adjustment to process the Baselines, the network coordinates were obtained in the 2015 2016,2017 and 2018 cycles. Conduct assessment of the change of milestones between 2 cycles 1-j (j- number of cycle):$T_U=dU/m_{dU}$, $T_N=dN/m_N$, $T_E=dE/m_{dE}$ .Check the Criteria (t-dítribution) (Ghilani, C., Wolf, P:,(2017)): $|T_U| < q_U$, $|T_N|<q_N$,$|T_E| < q_E$. If the test value is greater then the critical value, then there are significant deformations in the points.

Calculation results and checking statistical criteria can be seen that all 6 points have shifted.

3.2. Calculation of displacement parameters

In each observation period, calculations and adjustments are made according to formulas (32)-418). These results are used to calculate the displacement parameters 1-2,1-2-3,1-2-3-4 according to formulas (16) to (31). The calculation results are presented in the following tables (Round to mm).

Table 1. Displacement parameters dU,dN dE (mm) , v_U, v_N, v_E,a_U, a_N,a_E between 2015 and

	point 1	point 2	point 3	point 4	point 5	point 6	point 7	point 8
dU (mm)	51	-11	-15	28	-39	-4	12	-16
dN (mm)	0	0	0	0	0	0	0	0
dE (mm)	0	0	0	0	0	0	0	0
v_U (mm/yeear)	61	-13	--18	33	-47	--4	14	-20
v_N (mm/yeear)	0	0	0	0	0	0	0	0
v_E (mm/yeear)	0	0	0	0	0	0	0	0

Table 2.Displacement parameters dU,dN dE (mm) , v_U, v_N, v_E,a_U, a_N,a_E between 2015 ,2016 ,2017

	point 1	point 2	point 3	point 4	point 5	point 6	point 7	point 8
dU (mm)	11	-76	-47	-31	-46	-13	-20	-45
dN (mm)	0	0	0	0	0	0	0	0
dE (mm)	0	0	0	0	0	0	0	0
v_U (mm/yeear)	-86	-91	-40	-102	8	-12	-54	-34
v_N (mm/yeear)	0	0	0	0	0	0	0	0
v_E (mm/yeear)	0	0	0	0	0	0	0	0
a_U (mm/yeear^2)	-91	-53	-16	-86	32	-5	-45	-12
a_N (mm/yeear^2)	0	0	0	0	0	0	0	0
a_E (mm//yeear^2)	0	0	0	0	0	0	0	0

Table 3 Displacement parameters dU,dN dE (mm) , v_U, v_N, v_E,a_U, a_N,a_E between 2015 ,2016 ,2017and 2018

	point 1	point 2	point 3	point 4	point 5	point 6	point 7	point 8
dU (mm)	12	-31	-10	0	-33	-12	-8	-27
dN (mm)	0	0	0	0	0	0	0	0
dE (mm)	0	0	0	0	0	0	0	0
v_U (mm/yeear)	22	100	70	0	23	71	34	42
v_N (mm/yeear)	0	0	0	0	0	0	0	0
v_E (mm/yeear)	0	0	0	0	0	0	0	0
a_U (mm/yeear^2)	41	109	68	0	20	11	44	47
a_N (mm/yeear^2)	0	0	0	0	0	0	0	0
a_E (mm//yeear^2)	0	0	0	0	0	0	0	0

3.3. Discussion

From the calculated displacement parameters (from Table 1 to Table 3), it is clear that only the displacement of the U coordinate, i.e., related to uplift and subsidence, is present. The horizontal displacement is stable. Point 1 tends to uplift, while points 2, 3, 5, 6, 7, and 8 subside. Only point 4 up to cycle 4 shows a stable trend. This result demonstrates the advantage of adjusting the GNSS network in the local topocentric coordinate system in deformation analysis. If the correction were performed in the geocentric coordinate system, the picture would not be as clear.

An experiment was conducted using data from the first three cycles to predict the fourth cycle. The results showed that the largest deviation from the actual result was 80 mm.

4. CONCLUSION

This paper presents the theoretical basis for developing a recursive adjustment method to address the problem of assessing land displacement and deformation due to natural disasters or climate change in a dynamic model over time. To implement this solution, GNSS satellite technology was applied within the observation area. The monitoring network was individually calibrated in each measurement cycle. The aggregation and analysis of displacement were performed in the local topocentric coordinate system. The advantage of this method is that it allows for clear visualization of horizontal or vertical displacement (uplift or subsidence). Experimental results demonstrate the effectiveness of the method through the detection of vertical displacement of points in the monitoring network in central Vietnam.

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