How to Control Survey Quality when Estimation and Testing are Combined

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Key words: Detection-Identification-Adaptation, DIA-estimator, Estimation, Testing, Quality Control, DIA confidence regions

SUMMARY

Methods of surveying quality control consist generally of both parameter estimation and statistical hypothesis testing. Estimation, often via a least-squares adjustment, is then carried out to obtain optimal estimates of the parameters of interest, while statistical testing is conducted to detect and identify possible misspecifications in the assumed working hypothesis. Although estimation and testing are conducted on the same data, current quality control methods fail to take the interdependence between them into account. As one of the serious consequences, in particular for risk-critical applications, the resulting quality description will become too optimistic. The DIA (Detection, Identification, Adaptation)-method presented in this contribution avoids this serious pitfall through a rigorous integration of the probabilistic uncertainties of both estimation and testing. Various examples will be given to illustrate the DIA-methodology and underlying concepts involved.

SAMENVATTING

Methoden voor kwaliteitscontrole bij landmeetkundige metingen bestaan over het algemeen zowel uit parameter-schatting als statistische hypothese toetsing. Schatting, vaak via de kleinste-kwadraten-methode, wordt vervolgens uitgevoerd om optimale schattingen van de parameters van belang te verkrijgen, terwijl statistische toetsing wordt toegepast om mogelijke foutieve aannames in de veronderstelde werkhypothese te detecteren en te identificeren. Hoewel schatting en toetsing op dezelfde gegevens worden uitgevoerd, houden de huidige methoden voor kwaliteitscontrole geen rekening met de onderlinge afhankelijkheid daartussen. Eén van de gevolgen hiervan, met name voor risico kritische toepassingen, is dat de resulterende kwaliteitsbeschrijving te optimistisch zal zijn. De in deze bijdrage gepresenteerde DIA-methode (Detection, Identification, Adaptation) voorkomt dit ernstige probleem door een strenge integratie van de probabilistische onzekerheden van zowel schatting als toetsing. Verschillende voorbeelden worden gegeven om de DIA-methodologie en de onderliggende concepten te illustreren.

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1. INTRODUCTION

Surveying and geodetic practice has always combined *measurement*, *adjustment*, and *quality control*. In classical workflows, however, these elements are often treated as separate steps: observations are first adjusted, then residuals are inspected, and finally decisions are made about accepting, rejecting, or modifying data. The Detection–Identification–Adaptation (DIA) estimator [1-4] however, is built on a fundamentally different idea. Its central essence is that *estimation and statistical testing are not separate tasks, but inseparable components of a single decision process*. In the DIA framework, every estimate is conditional on a statistical decision, and every statistical test has direct consequences for the estimator that is ultimately adopted. This integration has far-reaching consequences for surveying. It clarifies what it means for an estimate to be "reliable," makes the role of significance levels explicit, and explains why different survey outcomes may arise from the same data under different decision strategies. This survey article places this core idea—*the unification of estimation and testing*—at the center of the discussion and shows its consequences for the control of survey quality.

2. THE DIA PHILOSOPHY: ESTIMATION *AS* A DECISION PROCESS

At the heart of the DIA estimator lies a simple but powerful principle: *one does not estimate parameters without first deciding which model is valid*. Conversely, one does not test a model without considering how the test outcome affects the final estimates. In surveying terms, the DIA estimator replaces the notion of a single adjustment result by a *set of conditional estimators*, each associated with a specific hypothesis about the data. The final estimate is selected only after a sequence of statistical decisions has been made.

This philosophy is operationalized through three tightly coupled steps:

i. **Detection** – deciding whether the assumed observation model is acceptable;
ii. **Identification** – deciding which alternative model best explains the data if it is not;
iii. **Adaptation** – adopting the estimator that is optimal for the selected model.

Crucially, these steps are not optional add-ons. They define the estimator itself. The DIA estimator is therefore a *rule*: a mapping from observations to decisions and, conditional on those decisions, to parameter estimates (see Figure 1).

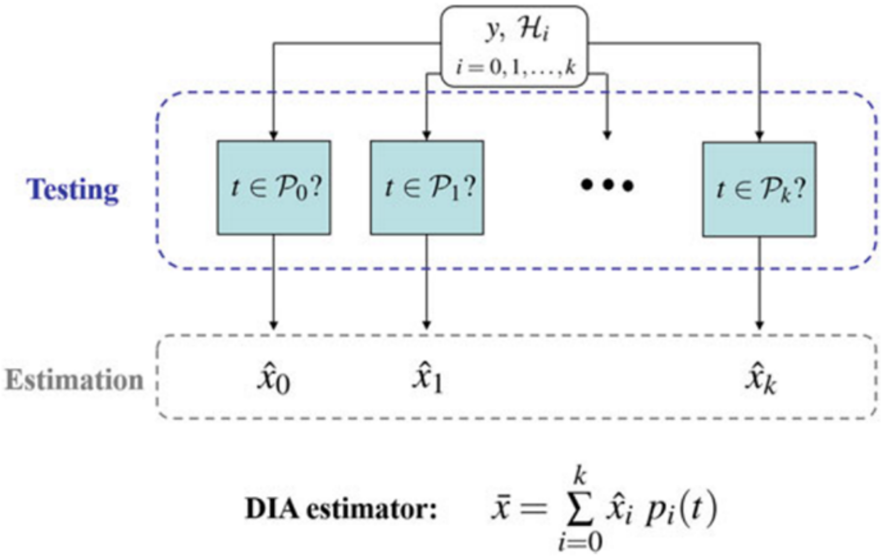

Fig 1: Estimation and testing combined leads to the DIA-estimator [1, 2, 5]. Vector of observations y, with null-hypothesis H_0 and k alternative hypotheses H_i. Misclosure (or residual) vector t resides in one of the regions P_i, which together form a partition of misclosure space. Hypothesis H_α is selected when t resides in P_α, in which case the least-squares solution $\hat{x}_\alpha$ of H_α is provided as output.

In the DIA framework, **detection** is the gateway to estimation. Before any estimate is accepted, the surveyor must decide whether the assumed model, the null-hypothesis H_0, is statistically consistent with the observations. Detection is carried out by a global hypothesis test of H_0. Its outcome determines whether the conventional least-squares estimator $\hat{x}_0$ under H_0 may be trusted or whether it must be rejected as being based on an invalid model.

Once detection indicates that the assumed model is inadequate, the surveyor is faced with a decision problem: which model should replace it? **Identification** answers this question. In the DIA framework, identification is not merely diagnostic. Selecting an alternative hypothesis directly determines which estimator will be used in the adaptation step. A wrong identification therefore does not only mislabel the error source; it leads to a different estimate.

This explicit link between identification and estimation highlights an important consequence of the DIA approach: *estimation uncertainty includes decision uncertainty.* Even when an error is detected, the reliability of the final estimate depends on the probability of correct identification—an aspect that is often implicit or ignored in classical practice.

Adaptation is the step where estimation and testing fully merge. The adapted estimator is conditional on the outcome of detection and identification. From a surveying perspective, adaptation corresponds to familiar actions such as deleting an observation or introducing a bias parameter. The DIA framework, however, makes explicit that these actions are not heuristic corrections but statistically justified consequences of earlier decisions. The key consequence is that *there is no single 'best' estimator independent of testing.* Instead, the DIA approach yields a family of estimators, each optimal under its corresponding hypothesis. The DIA estimator $\bar{x}$ itself is therefore a composite combination, which can be captured with the compact formula

$$\bar{x} = \sum_{j=0}^{k} \hat{x}_j \; p_j(t)$$

in which $p_j(t)$ is the indicator function of region P_j, i.e. $p_j(t) = 1$ when t resides in P_j and $p_j(t) = 0$ elsewhere (see Figure 1).

3. HOW TO JUDGE THE QUALITY OF THE DIA ESTIMATOR: A PDF-VIEW

The most fundamental way to describe and judge the quality of a DIA estimator is through its probability density function (PDF). Unlike classical estimators, whose PDF is typically a single multivariate normal distribution, the DIA estimator has a composite PDF that reflects both estimation uncertainty and decision uncertainty.

The DIA PDF $f_{\bar{x}}(x)$ reveals an essential consequence of integrating testing and estimation: even when all conditional estimators are unbiased and normally distributed, the overall DIA estimator $\bar{x}$ need not be normally distributed. Its PDF may be multimodal, asymmetric, or heavy-tailed, reflecting the possibility of incorrect decisions. *These features are not deficiencies; they are faithful representations of the true uncertainty faced by the surveyor*. Judging the quality of a DIA estimator therefore means judging the shape and mass distribution of its PDF.

As two such examples under H_0, consider Figure 2 [6, 7]. It shows in its top row two skyplots of GPS satellites viewed from Melbourne, Australia, while the bottom row shows the contour plots of their horizontal positioning DIA-PDF, with position-estimation based on pseudorange SPP and testing on outlier datasnooping, assuming σ = 50 cm and 90% probability of H_0 correct acceptance. The contour lines are non-ellipsoidal, thus illustrating how the PDF of the DIA-estimator differs from the typical Gaussian distributions of the classical estimators. Their shapes reflect how possible incorrect decisions of outlier testing propagate into the horizontal positioning domain.

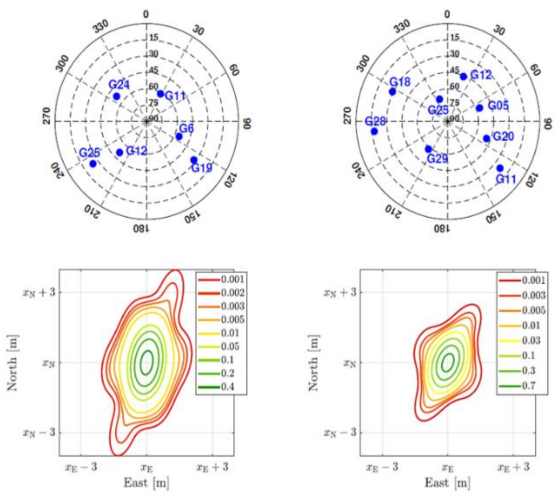

Fig 2: Two Melbourne skyplots with the contour lines of their data snooping based, horizontal positioning DIA-PDF.

Viewing the DIA estimator through its PDF leads to a clear and rigorous criterion for quality assessment:

Key Concept

The quality of a DIA estimator is the probability that its outcomes lie sufficiently close to the true parameter value, evaluated with respect to its full (mixture) probability density function.

This means that estimator quality is judged by *how much probability mass is concentrated near the true value*, not merely by the size of a covariance matrix. This criterion provides a transparent basis for comparing different survey designs, testing strategies, and decision thresholds. It also explains why significance levels, redundancy, and network geometry directly influence estimator quality. In this sense, the PDF of the DIA estimator is not merely a theoretical construct - *it is the most complete and honest description of the uncertainty inherent in high-precision surveying*.

The probability-of-closeness interpretation of DIA quality also provides a direct conceptual bridge to integrity monitoring concepts such as *integrity risk* and *protection levels*, which are increasingly important in GNSS and safety-critical surveying applications. Integrity risk can be interpreted as the probability that the estimation error exceeds a specified alert limit. In DIA terms, this is simply the complement of a probability-of-closeness statement evaluated with respect to the DIA PDF. Similarly, a protection level can be understood as the smallest tolerance radius for which the probability of closeness exceeds a prescribed integrity requirement.

This shows that integrity monitoring is not an external add-on to estimation, but a natural consequence of the DIA framework. The DIA estimator provides the probabilistic machinery needed to assess not only accuracy, but also trustworthiness—using a single, coherent statistical description.

4. DIA vs CLASSICAL CONFIDENCE REGIONS

Confidence regions play a central role when quantifying the probabilistic closeness between an estimator and the unknown true parameter vector. Classical confidence regions are rooted in estimation theory under the assumption of a fixed and correct model, so that uncertainty arises solely from observation noise.

Classical confidence regions are typically constructed as ellipsoids of the form

$$C_\alpha = \{x\colon\ (\hat{x}_0 - x)^T Q_{\hat{x}_0\hat{x}_0}^{-1} (\hat{x}_0 - x)\ \leq\ c_\alpha\}$$

where $\hat{x}_0$ is a fixed estimator, $Q_{\hat{x}_0\hat{x}_0}$ its variance-covariance matrix, and c_α a quantile ensuring coverage probability $1-\alpha$. These regions rely on implicit assumptions:

- That the estimator form is fixed and non-random,
- That the error distribution is unimodal (often Gaussian),
- That the underlying model or hypothesis is correct.

Hence, classical confidence regions quantify *conditional uncertainty*: they describe estimator accuracy assuming that the adopted model and estimator are correct. The DIA estimator however, includes the crucial uncertainty of adoption as well. The estimator is therefore random not only because of observation noise, but also because its functional form depends on the testing outcome. This implies that classical confidence regions fail to do proper justice to the quality of surveying outcomes and that instead the full PDF $f_{\bar{x}}(x)$ of the DIA estimator is required for constructing honest confidence regions. DIA confidence regions are therefore constructed as *highest-density regions* of the DIA PDF:

$$C_\alpha^{DIA} = \{x:\ f_{\bar{x}}(x)\ \leq\ \tau_\alpha\}$$

with the threshold τ_α determined such that a coverage probability of $1-\alpha$ is ensured. These regions are optimal in the sense that they have the smallest volume among all regions having coverage probability $1-\alpha$ [1, 2]. A detailed numerical algorithm for computing these confidence regions can be found in [6].

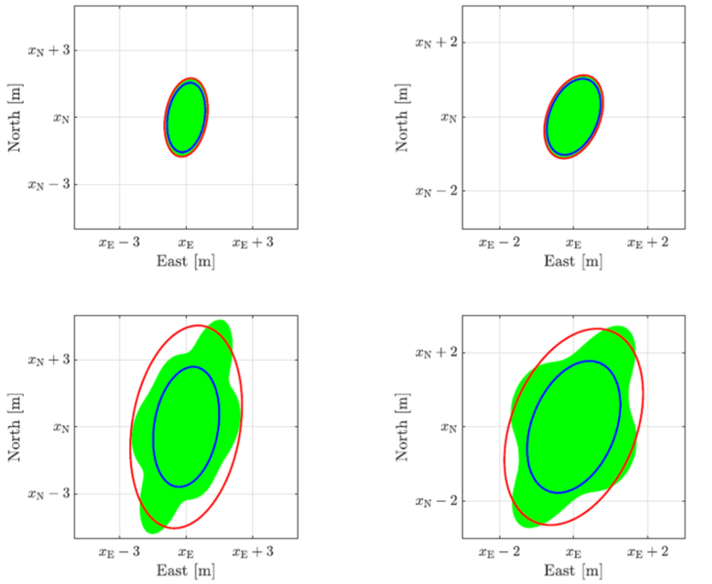

Fig 3: Illustration of confidence regions for the two skyplots of Figure 2, with α = 0.1 (top) and α = 0.001 (bottom): C_α^{DIA} is indicated by the green area and C_α by the blue ellipse.

Figure 3 shows the 100(1−α)% confidence regions for the two skyplots of Figure 2, with α = 0.1 (top) and α = 0.001 (bottom). The DIA confidence region C_α^{DIA} is indicated by the green area and the classical confidence region C_α by the blue ellipse. Note the difference in size and shape between C_α^{DIA} and C_α, in particular when higher confidence is required (smaller α), such as in safety-critical and liability-critical applications.
The red ellipse describes the *ellipsoidal* uncertainty of the DIA estimator. It is the 100(1−α)% coverage area of the DIA estimator when its shape is forced to be that of

the classical confidence region. As the blue ellipse will always lie inside the red one, their difference visualizes by how much the classical confidence region provides a *too optimistic* quality description of the surveyor's positioning results.

The key differences between the DIA- and classical confidence regions are therefore as follows:

- Conditional vs unconditional: Classical regions provide conditional coverage, given a fixed estimator or hypothesis, while DIA regions provide unconditional coverage, averaged over all possible testing outcomes. This makes DIA confidence regions honest in the sense that they do not hide the risk of incorrect decisions.
- Geometry and shape: Classical confidence regions are typically ellipsoidal and convex. In contrast, DIA confidence regions can be nonconvex or even disconnected [6, 7]. Their geometric features directly visualize the interaction between testing and estimation. As they reveal how uncertainty mass is redistributed due to testing, they allow practitioners to see the impact of decision errors spatially.
- Interpretation of Conservatism: Classical regions are too optimistic as they ignore the testing decisions. DIA regions, by contrast, may be larger or more complex, not because they are conservative, but because they represent all relevant sources of uncertainty.

DIA confidence regions thus extend classical confidence regions by explicitly incorporating detection and estimator adaptation into parameter uncertainty quantification. They characterize not merely the precision of an estimator under a fixed model, but the uncertainty of the *entire detection–identification–adaptation chain*. They are therefore essential for rigorous and honest performance assessments.

5. CONCLUSIONS

The Detection–Identification–Adaptation (DIA) estimator formalizes surveyor's quality control as an integral part of the estimation problem itself, rather than as a separate, sequential procedure. In contrast to the classical approach, the DIA framework embeds testing, decision making, and model modification directly into the estimation structure.
The classical separation between testing and estimation obscures the interaction between estimator precision, test power, redundancy, and decision thresholds. The DIA estimator overcomes this limitation by explicitly linking estimation and testing through a common statistical foundation. Within this integrated framework, detection determines whether the assumed model is statistically admissible, identification localizes the source of inconsistency within a structured set of alternative hypotheses, and adaptation updates the model and estimator accordingly. These steps are not ad hoc corrections but part of a closed-loop process in which estimation and testing continuously inform each other. As a result, the effects of adaptation on both parameter estimates and subsequent test performance are explicitly accounted for.

By unifying estimation and testing, the DIA concept [1, 2] provides a quantitative description of reliability that goes beyond classical accuracy measures. It enables surveyors to evaluate the probability of correct detection and identification, the risk of missed errors, and the impact of quality control decisions on the final parameter estimates of interest. This makes fit-for-purpose reliability a designable and analyzable property of the surveying system, rather than a posterior diagnostic. For modern surveying and GNSS applications, often associated with increased reliability requirements, this integrated perspective is essential.

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BIOGRAPHICAL NOTES

Peter is Vice-President of the International Association of Geodesy (IAG), a Professor of Geodesy at Delft University of Technology, and an elected member of the Royal Netherlands Academy of Arts and Sciences. His past academic positions include Head of the Delft Earth Observation Institute, Science Director of the Australian Centre for Spatial Information and Federation Fellow of the Australian Research Council. He has been research-active in various fields of Geodesy, with current research focused on the development of theory, models, and algorithms for high-accuracy applications of satellite navigation and remote sensing systems. His scientific contributions have been recognized through various awards, including the IAG Bomford Prize, the Humboldt Research Award, the ION Kepler Award, and EGU's Vening-Meinesz Medal. He has an Honorary Doctorate from CAS and is an elected Fellow of IUGG, IAG, UK-RIN, and USA-ION.

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