Single Frequency Precise Point Positioning Using GPS and Galileo Observables

Akram AFIFI and Ahmed EL-RABBANY, Canada

Key words: PPP, GPS, Galileo, Stochastic Characteristics

SUMMARY

This paper develops a precise point positioning (PPP) model for combined GPS/Galileo single-frequency observations. Although improves the solution availability and accuracy, combining GPS and Galileo observables introduces additional biases that must be modelled. These include the GPS-to-Galileo time offset and the inter-system bias. Additionally, to take full advantage of the Galileo E1 signal, it is essential that its stochastic characteristics are rigorously modelled. In this paper, various sets of GPS and Galileo measurements collected at two stations with short separation were used to investigate the stochastic characteristics of Galileo E1 signal. As a by-product, the stochastic characteristics of the legacy GPS P1 code was obtained and then used to verify the developed stochastic model of the Galileo signal. It is shown that sub-decimeter level accuracy is possible through our single-frequency GPS/Galileo PPP model. As well, the addition of Galileo improves the PPP solution convergence by about 30% in comparison with GPS-only solution.

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1. INTRODUCTION

Dual-frequency GPS PPP technique has been proven to be capable of providing positioning solution at the sub-decimeter level in static mode. This is achieved through rigorous modeling or estimation of all errors and biases. Unfortunately, dual-frequency GPS receivers may not provide a cost effective solution to many users. In addition, a drawback of a single GNSS system such as GPS is the availability of sufficient number of visible satellites in urban areas. Galileo satellite system offers additional visible satellites to the user, which is expected to enhance the satellite geometry and the overall PPP solution when combined with GPS (Hofmann-Wellenhof et al., 2008). As shown in Afifi and El-Rabbany (2013), combining GPS and Galileo observations in a PPP solution enhances the positioning solution convergence and precision in comparison with GPS-only PPP solution. This, however, requires rigorous modelling of all errors and biases.

Generally, the mathematical model for GNSS PPP consists of two parts, namely functional and stochastic models. The functional part describes the physical or geometrical characteristics of the parameters of the PPP model, while the stochastic part describes the statistical (or stochastic) properties of the residual components in the functional model. The stochastic model is represented by the observations variances-covariance parameters (i.e., the covariance matrix). Functional models related to the GNSS observables have been extensively studied by many researchers. This, however, is not the case with the more complex stochastic models. Often, a simplified empirical stochastic model is used in GNSS positioning, which assumes that all the GNSS observables are statistically independent and of the same quality. This, however, can lead to an overestimation of the PPP parameters.

This research aims to improve the single-frequency GPS/Galileo PPP solution through rigorous modelling of the stochastic characteristics of Galileo signal. In addition, this research studies the effect of satellite elevation angle variation on the stochastic characteristics of the GNSS observables. A receiver system noise test is performed to determine the stochastic characteristics of Galileo E1 and E5a and GPS L1 signals, respectively. To verify the obtained results, the newly developed stochastic model is implemented to assess the effect of the stochastic characteristics on the combined GPS/Galileo PPP solution precision and convergence time. The results of the combined GPS/Galileo solution show an improvement of up to 30% in the solution convergence time and a positioning accuracy at the sub-decimeter level.

2. GPS AND GALILEO OBSERVATION MODELS

GNSS observations are affected by random and systematic errors which should be considered to obtain accurate positioning. The accuracy of precise point positioning depends on the ability to mitigate all kinds of errors. These errors can be categorized into three classes, satellite related errors, signal propagation related errors, and receiver/antenna configuration errors. GNSS errors attributed to the satellites, include satellite clock errors, orbital errors, satellite hardware delay, satellite antenna phase centre variation, and satellite initial phase bias. Errors attributed to signal propagation, include the delays of the GNSS signal as it passes through the ionospheric and tropospheric layers. Errors attributed to receiver/antenna configuration include the receiver clock errors, multipath error, receiver noise, receiver hardware delay, receiver initial phase bias, and receiver antenna phase center variations (El-Rabbany, 2006).

In addition to the above errors, combining GPS and Galileo observations in a PPP model introduces additional errors such as GGTO due to the fact that each system uses a different time frame. GPS system uses the GPS time system which is referenced to coordinated universal time (UTC) as maintained by the US Naval Observatory (USNO). On the other hand, Galileo satellite system has its time frame namely The Galileo system time (GST), which is a continuous atomic time scale with a nominal constant offset with respect to the international atomic time (TAI) (Hofmann-Wellenhof et al., 2008). As well, GPS and Galileo observations are on different reference frames, which should be considered in the combined PPP model. Taking the above errors and biases into consideration, the GPS and Galileo observation equations can be written as:

$$\mathrm{P_G} = \rho_G(t_G,(t-\tau)_G) + \mathrm{c}[\mathrm{dt_r}(t_G) - \mathrm{dt^s}(t-\tau)_G] + T_G + I_G + c[d_r(t_G) + d^s(t-\tau)_G] + d_{mp} + e_{PG} \tag{1}$$

$$\mathrm{P_E} = \rho_E(t_E,(t-\tau)_E) + \mathrm{c}[\mathrm{dt_r}(t_E) - \mathrm{dt^s}(t-\tau)_E] + T_E + I_E + c[d_r(t_E) + d^s(t-\tau)_E] + d_{mp} + e_{PE} \tag{2}$$

$$\begin{aligned}\Phi_\mathrm{G} = \rho_G(t_G,(t-\tau)_G) &+ \mathrm{c}[\mathrm{dt_r}(t_G) - \mathrm{dt^s}(t-\tau)_G] + T_G - I_G + \\ &c[\delta_r(t_G) + \delta^s(t-\tau)_G] + \lambda[N_G + \phi_r(t_0) - \phi^s(t_0)] + \delta_{mp} + \varepsilon_{\varphi G}\end{aligned} \tag{3}$$

$$\begin{aligned}\Phi_\mathrm{E} = \rho_E(t_E,(t-\tau)_E) &+ \mathrm{c}[\mathrm{dt_r}(t_E) - \mathrm{dt^s}(t-\tau)_E] + T_E - I_E + \\ &c[\delta_r(t_E) + \delta^s(t-\tau)_E] + \lambda[N_E + \phi_r(t_0) - \phi^s(t_0)] + \delta_{mp} + \varepsilon_{\varphi E}\end{aligned} \tag{4}$$

where the subscript *G* refers to the GPS satellite system and the subscript *E* refers to the Galileo satellite system; P_G and P_E are pseudoranges for the GPS and Galileo systems, respectively; Φ_G and Φ_E are the carrier phase measurements of the GPS and Galileo systems, respectively; $dt_r(t)$, $dt^s(t-\tau)$ are the clock error for receiver at reception time t and satellite at transmitting time t-τ, respectively; $d_r(t)$, $d^s(t-\tau)$ are frequency dependent code hardware delay for receiver at reception time t and satellite at transmitting time t-τ, respectively; $\delta_r(t)$, $\delta^s(t-\tau)$ are frequency-dependent carrier phase hardware delay for receiver at reception time t and satellite at transmitting time t-τ, respectively; T is the tropospheric delay; I is ionospheric delay; d_{mp} is code multipath effect; δ_{mp} is the carrier phase multipath effect; λ is the wavelengths of carrier frequencies, respectively; $\Phi_r(t_0)$, $\Phi^s(t_0)$ are frequency-dependent initial fractional phases in the receiver and satellite channels; N is the integer number of cycles for the carrier phase measurements, respectively; c is the speed of light in vacuum; and ρ is the true geometric range from receiver at reception time to satellite at transmission time; e_P, ε_Φ are the relevant noise and un-modeled errors.

Several organizations such as the International GNSS Service (IGS) and the Cooperative Network for GIOVE Observations (CONGO) network provide the user with precise products, including precise satellite orbit and clock corrections. IGS precise satellite orbit and clock corrections contain the satellite hardware delay of the ionosphere-free linear combination of GPS L1 and L2 signals (Kouba, 2009). On the other hand, CONGO satellite precise orbital and clock corrections include the satellite hardware delay of the ionosphere-free linear combination of Galileo E1 and E5a signals (Montenbruck at al., 2009). In this research, the precise orbit and satellite clock corrections from the CONGO network are used for both GPS and Galileo satellites. In addition, the GPS receiver hardware delay is lumped to the receiver clock error. This, in turn introduces a new term in the Galileo observation equations, which represents the difference between the satellite hardware delays of GPS and Galileo signals. A new unknown (ISB) is considered in our model to account for the system time offset as well as the new satellite hardware difference term as shown in equations 7 and 8. The receiver and satellite hardware delays can be lumped to the receiver clock error and to the GGTO as all of these errors are timing errors. Equations 5 to 8 show the final combined GPS and Galileo PPP model.

$$\mathrm{P_G} = \rho_G + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s_{IGS}}] + T_G + I_G + e_{PG} \tag{5}$$

$$\mathbf{\Phi}_\mathrm{G} = \rho_G + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s_{IGS}}] + T_G - I_G + \lambda\tilde{N}_G + \varepsilon_{\varphi G} \tag{6}$$

$$\mathrm{P_E} = \rho_E + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s_{CON}}] + ISB + T_E + I_E + e_{PE} \tag{7}$$

$$\mathbf{\Phi}_\mathrm{E} = \rho_E + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s_{CON}}] + ISB + T_E - I_E + \lambda\tilde{N}_E + \varepsilon_{\varphi E} \tag{8}$$

where $\tilde{N}$ is the ambiguity parameter including frequency-dependent initial fractional phases in the receiver and satellite channels; *ISB* is the newly introduced unknown parameter.

3. STOCHASTIC MODEL DEVELOPMENT

The receiver measurement noise results from the limitations of the receiver's electronics and can be determined through receiver calibration or test. Two tests are usually carried out to determine the system noise level, namely the zero and short baselines tests. The zero baseline test employs one antenna followed by a signal splitter that feeds two or more GPS receivers. Using the zero baseline test, several receiver problems can be investigated, such as interchannel biases and cycle slips. The single antenna cancels out the real world systematic problems such as multipath in addition to the preamplifier's noise. The short baseline test, on the other hand, uses two receivers a few meters apart and is usually carried out over two consecutive days. In this case, the double difference residuals of one day would contain the system noise and the multipath effect. As the multipath effect repeats almost every day for GPS system, differencing the double difference residuals of the two consecutive days cancels out the multipath effect and leaves the scaled system noise (El-Rabbany, 2006). However, multipath effect is not repeatable for the Galileo satellite system as the satellites take about 14 hours 4 minutes 41 seconds to orbit the Earth (Hofmann-Wellenhof et al., 2008).

In this research, a short baseline test is used to determine the stochastic characteristics of the E1 signal, assuming that multipath does not exist. Usually, this test is performed using the same type of receivers. Unfortunately, in this research, two different receivers were available (Septentrio POLARX4TR and Trimple NETR9) for the test, which can observe the Galileo measurements. This, however, were considered when processing the data as shown in the sequel. The pseudorange and carrier phase equations can be re-written as, assuming no multipath and dropping the time argument:

$$\mathrm{P_i} = \rho + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s}]_\mathrm{i} + c[d_r + d^s]_i + T_i + I_i + e_{Pi} \tag{9}$$

$$\Phi_\mathrm{i} = \rho + \mathrm{c}[\mathrm{dt_r} - \mathrm{dt^s}]_\mathrm{i} + c[\delta_r + \delta^s]_i + T_i - I_i + \lambda\widetilde{N} + e_{\Phi i} \tag{10}$$

Differencing the pseudorange and carrier phase equations of each receiver cancels out the geometric term, satellite and receiver clock error, and tropospheric delays, as shown in Equations (120) and (21). The remaining terms include the satellite and receiver hardware delays, ionosphere delay, the ambiguity parameter and the system noise.

$$\Delta R_1 = P_{\mathrm{R1}} - \Phi_{\mathrm{R1}} = \mathrm{c}[\mathrm{d_r} - \mathrm{d^s}]_1 + c[\delta_r - \delta^s]_1 + \Delta\lambda\widetilde{N}_1 + 2I + e_P \tag{11}$$

$$\Delta R_2 = P_{R2} - \Phi_{R2} = \mathrm{c}[\mathrm{d_r} - \mathrm{d^s}]_2 + c[\delta_r - \delta^s]_2 + \Delta\lambda\widetilde{N}_2 + 2I + e_p \tag{12}$$

It should be pointed out that the noise parameters in Equations 20 and 21 are essentially those of the pseudorange measurements. The phase measurement noise has been neglected due to its small size compared to that of the pseudorange measurements (Elsobeiey and El-Rabbany, 2010). The receiver hardware delay is assumed to be stable over the observation period (four hours in this research), while the ambiguity parameter and initial phase bias are constants for a continuous session of measurements. As such, they can be removed from the model through differencing with respect to the first value of the series. Using this approach, only the differenced system noise remains in the model.

In PPP, most of existing observation stochastic models are empirical functions such as sine, cosine, exponential and polynomial functions. Most of these stochastic models are functions of the satellite elevation angles (Leandro and Santos 2007). Unfortunately, existing stochastic models may not be valid for all receiver types and GNSS signal frequencies. As such, it is essential that new stochastic models are developed for the Galileo signal. The data series developed are divided into nine bins depending on the satellite elevation angle, starting from 0° to 90° with increments of 10° (i.e., 0º to 10º, 10º to 20º, etc.). The standard deviation of the differenced system noise for each bin is estimated. A least squares regression analysis is performed to obtain the best-fit model of the estimated standard deviations. Three empirical functions were tested for this purpose, namely an exponential, a polynomial and a rational model as shown in Table 1. The best-fit model is selected based on the goodness of fit test, i.e., the one with the largest R2 (R-squared) statistic (Draper, 2002).

Table 1 Summary results of regression fitting functions with 95% confidence level

	Exponential function			Polynomial function			Rational function		
	$STD = \mathrm{a} \times e^{(-\mathrm{b} \times ELE)} + \mathrm{c}$			$STD = -\mathrm{a} \times ELE^3 + \mathrm{b} \times ELE^2 - \mathrm{c} \times ELE + \mathrm{d}$			$STD = \frac{(a \times ELE^2 - b \times ELE + c)}{(ELE + d)}$		
	E1	E5a	L1	E1	E5a	L1	E1	E5a	L1
a	0.6383	0.3692	0.6830	1.835e-6	5.892e-6	1.473e-6	3.5e-3	4.315e-3	6.087e-3
b	0.0763	0.0753	0.0730	3.688e-4	1.445e-4	3.195e-4	0.2703	0.5155	0.6533
c	0.2150	0.0974	0.1751	0.02443	0.01556	0.0228	22.93	28.36	36.57
d	-	-	-	0.7557	0.4014	0.7156	28.35	69.29	49.69
R^2	0.9995	0.9993	0.9994	0.9988	0.9977	0.9990	0.9990	0.9977	0.9984

where *ELE* is the satellite elevation angle in degrees; *STD* is the observation standard deviation

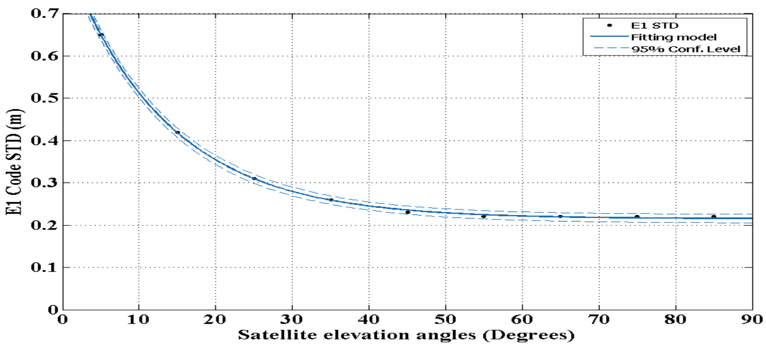

Figure 1 the standard deviation of Galileo E1 signal using exponential fit model

Figure 1 shows the variation of the stochastic characteristics of the Galileo E1 signal with respect to the satellite elevation angles. In addition, Figure 1 shows the results of the regression analysis using the exponential model with the 95% confidence level limits. The stochastic characteristic of the Galileo E1 signal is essentially constant above the elevation angle of 45°.

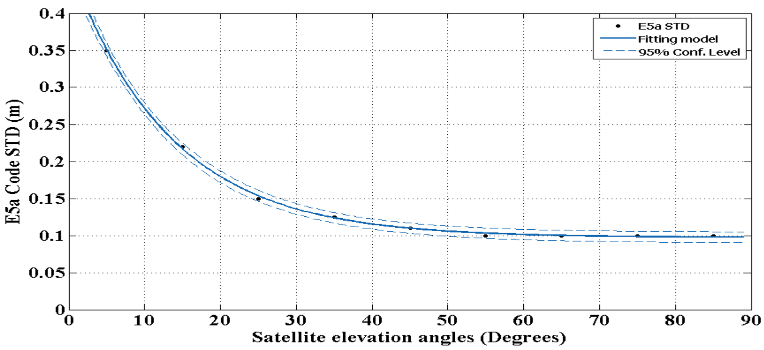

Figure 2 the standard deviation of Galileo E5a signal using exponential fit model

Figure 2 shows the variation of the standard deviation of the Galileo E5a signal with respect to the satellite elevation angles. In addition, Figure 2 shows the results of the regression analysis using the exponential model with the 95% confidence level limits.

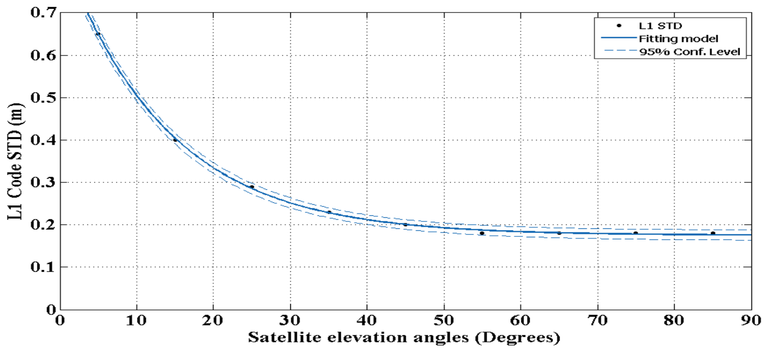

Figure 3 the standard deviation of GPS L1 signal using exponential fit model

Figure 3 shows the variation of the standard deviation of the GPS L1 signal with respect to the satellite elevation angles. In addition, Figure 3 shows the results of the regression analysis using the exponential model with the 95% confidence level limits.

4. RESULTS AND DISCUSSION

To test our PPP model and to verify the determined stochastic models of the Galileo E1 signal, Natural Resources Canada (NRCan) GPSPace PPP software was modified to handle the Galileo observations in addition to the newly developed stochastic models. The GPS/Galileo PPP solution is also obtained using an existing empirical function, namely the sine function, which is compared with the PPP solution obtained with newly developed stochastic model. Four stations were used to verify our PPP model, two stations in North America (UNB and USN) and two in Europe (Delft and GOP).

The IGS global ionospheric maps (GIM) product is used to correct for the ionospheric delay

(Schaer et al. 1998). Hopfield model has been used along with the Vienna mapping function as shown in Boehm and Schuh (2004) to correct the tropospheric delay (Hopfield, 1972). CONGO network precise satellite orbit and clock corrections are used for both GPS and Galileo satellites. Unfortunately, precise orbit and clock data from a single GPS/Galileo network solution were not available, which might create a small bias in the PPP results. Only the results of station UNB (North America) are presented in this paper. Similar results were obtained for other stations. The results of single frequency GPS PPP solution and the single-frequency GPS/Galileo PPP solution are obtained using two stochastic models, namely the sine function and the newly developed exponential function. Figure 4 shows the positioning results of the GPS-only PPP solution using the sine function as a representation of the observations stochastic model.

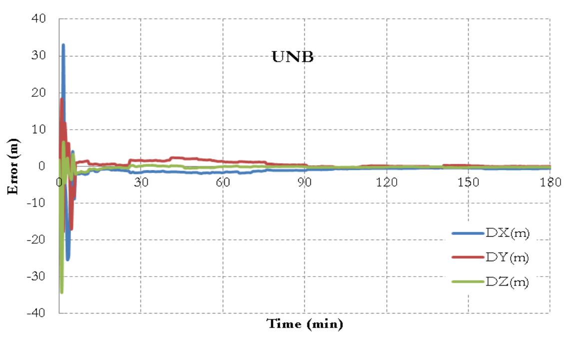

Figure 4 GPS PPP results using sine function stochastic model

As shown in Figure 4, the accuracy of the PPP solution with the GPS L1 signal is at the meter level. In contrast, when the newly developed exponential function is used, the single-frequency GPS PPP accuracy is improved to decimetre level (Figure 5).

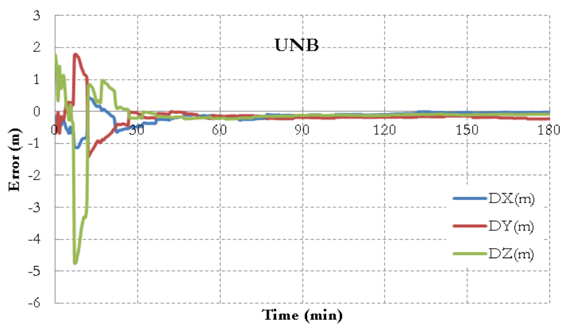

Figure 5 GPS PPP results using the newly developed stochastic model

Figures 6 and 7 show the PPP results of the combined GPS/Galileo observations with the sine and exponential functions, respectively.

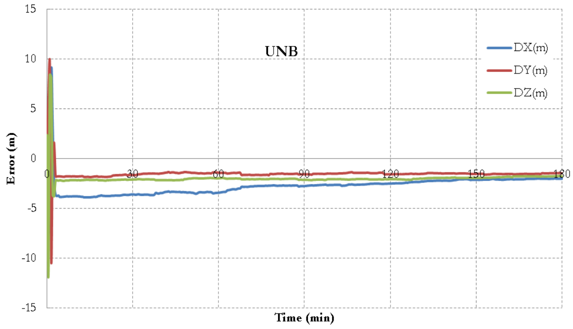

Figure 6 GPS/Galileo PPP results using empirical sine function stochastic model

As can be seen in Figure 6, the results of the GPS/Galileo PPP with the sine function show decimetre-level accuracy; however the solution convergences to this accuracy level after about three hours. Figure 7 shows that, when the exponential function is used, the solution converges to decimetre-level after 30 minutes or less. This is considered significant improvement, especially with single-frequency observations.

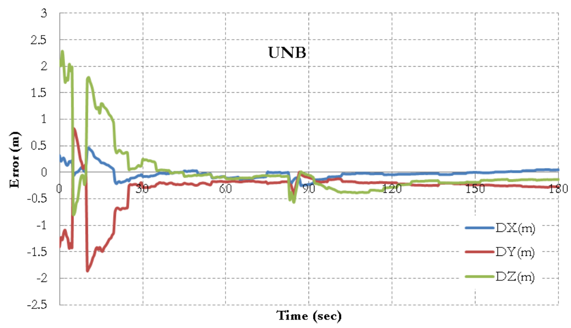

Figure 7 GPS/Galileo PPP results using the newly developed stochastic model

5. CONCLUSIONS

A PPP model has been introduced in this paper, which combines GPS and Galileo system observations. In addition, new stochastic models of Galileo E1 and E5a signals have been developed in this research. Three empirical functions are considered, namely exponential, polynomial and rational functions. Of the three models considered, the exponential function gave the best fit, based on regression analysis. As well, it has been shown that a sub-decimetre positioning accuracy is attainable with single-frequency GPS/Galileo PPP when the newly developed stochastic model is used. Furthermore, the solution convergence time has been reduced to less than 30 minutes, which represents a significant improvement for single-frequency observables.

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BIOGRAPHICAL NOTES

Prof. Ahmed El-Rabbany

Dr. Ahmed El-Rabbany obtained his PhD degree in GPS Satellite Navigation from the Department of Geodesy and Geomatics Engineering, University of New Brunswick, Canada. He is currently a full professor and Graduate Program Director at Ryerson University, Toronto, Canada. He also holds an Honorary Research Associate position at the University of New Brunswick. Dr. El-Rabbany's areas of expertise include satellite navigation, integrated navigation systems and hydrographic surveying. He is an associate editor of Geomatica and editorial board member for the Journal of Navigation. He also holds the position of President-Elect of the Canadian Institute of Geomatics. Dr. El-Rabbany received numerous awards in recognition of his academic achievements, including three merit awards and distinguished service award from Ryerson University and best papers and posters at various conferences and professional events. He was also honoured by a number of academic and professional institutions worldwide.

Mr. Akram Afifi

Mr. Akram Afifi is a PhD candidate in the Department of Civil Engineering, Ryerson University, Ontario, Canada. He holds the position of President of the student affairs committee with the Canadian Institute of Geomatics.

CONTACTS

Mr Akram Afifi
Ryerson University
350 Victoria Street
Toronto
COUNTRY CANADA
Tel. + 416-979-500 Ext 4623
Email: akram.afifi@ryerson.ca
Web site: Ryerson.ca

Prof Ahmed El-Rabbany
Ryerson University
350 Victoria Street
Toronto
COUNTRY CANADA
Tel. + 416-979-500 Ext 6472
Email: akram.afifi@ryerson.ca
Web site: Ryerson.ca