

## Overview

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## Introduction

The significance of road projects have been increasing day by day.
Because of

- Depending on social, cultural and economic developments, transportation requirements are increasing,
- As a result of technological developments, vehicles becoming faster and more comfortable,
- Parallel to all developments, the value of time is further increasing.

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## Introduction

- Horizontal and vertical curves used in transportation systems are the critical sections on the alignment.
- On the horizontal curve, lateral acceleration formed by centrifugal force adversely affects the road safety and reduces vehicle travel comfort.
- Evaluation criterion of the alignment geometry in terms of comfort is Jerk.
- During the curve design, to take into account of Jerk criterion is extremely important.



## Introduction

- In this study as distinct from other studies, minimum horizontal curve radius is derived using the limit value of lateral Jerk.
- The equations of minimum curve radius related to lateral Jerk are derived by analyzing the equations of lateral J erk in the literature.
- Minimum curve radius are calculated by using the equations derived.
- Calculated minimum curve radius are compared with the other minimum curve radius which were calculated with respect to different criterion.



## Determination of Minimum Horizontal Curve Radius

- For a given speed, the curve with the smallest radius is also the curve that requires the most centripetal force.

- Minimum curve radius must be selected very well for safe and comfortable driving in the alignment design.


Minimum Horizontal Curve-Radius based on the limit Value of Superelevation

- Superelevation is inclined roadway crosssection that employs the weight of a vehicle in the generation of the necessary centripetal force for curve negotiation.



## Minimum Horizontal Curve-Radius based on the limit Value of Superelevation

- Minimum curve radius can be derived to take into account the limit value of superelevation.
- Equation of minimum horizontal curve radius depend on the superelevation value is given by American Association of State Highway and Transportation Officials' A Policy on Geometric Design of Highways and Streets (AASHTO Green Book, 2001) as follows:

$$
\mathrm{R}_{\min }=\frac{\mathrm{V}^{2}}{127\left(0,01 \mathrm{e}_{\max }+\mathrm{f}_{\max }\right)}
$$



## Minimum Horizontal-Curve Radius based-on the limit

 Value of Superelevation- For railways; equation of minimum curve radius based on the superelevation was expressed as follows: (Baykal,2009).

$$
\mathrm{R}_{\min }=\frac{\sqrt{\mathrm{b}^{2}-\mathrm{u}_{\max }^{2}} \mathrm{~V}_{0}^{2}}{127,14 \mathrm{u}_{\max }}
$$

$\mathrm{R}_{\text {min }}$ : Minimum curve radius (m)
V : Velocity (km/s)
$\mathrm{e}_{\text {max }}$ : Maximum superelevation
$f_{\max }$ : Maximum allowable side friction factor
$\mathrm{u}_{\text {max }}$ : Maximum superelevation(m)
$\mathrm{V}_{\mathrm{o}}$ : The average speed ( $\mathrm{km} / \mathrm{h}$ )

$\mathrm{b} \quad:$ Width of the railway line $(\mathrm{m})$


## Minimum Horizontal Curve Radius based-on the Limit Value of Lateral Acceleration

- The change of velocity with respect to time is called acceleration.
- The lateral acceleration is created by centrifugal force.
- Minimum curve radius can be derived to take into account the limit value of the lateral acceleration. It was derived for the horizontal geometry as follows: (Baykal, 2009, p.351)

For highways: $\quad \mathrm{R}_{\min }=\frac{\mathrm{V}^{2}}{12,96\left(\sqrt{1+\mathrm{e}_{\max }^{2}} \mathrm{y}_{\mathrm{Y}}+\mathrm{e}_{\max } \mathrm{g}\right)}$

For railways: $\quad \mathrm{R}_{\min }=\frac{\sqrt{\mathrm{b}^{2}-\mathrm{u}_{\max }^{2}} \mathrm{~V}^{2}}{12,96\left(\mathrm{~b} \mathrm{a}_{\mathrm{Y}}+\mathrm{u}_{\max } \mathrm{g}\right)}$

Minimum Horizontal Curve Radius based on the Limit Value of Lateral J erk

- The change of the acceleration with respect to time is called Jerk.
- The concept of Jerk is defined as the third derivative of distance
- The Jerk is also called the second acceleration.
- In terms of road safety and comfort, Jerk is an important design criterion.
- Jerk is known as comfort criterion in the alignment design.


## Minimum Horizontal Curve Radius based on the Limit

 Value of Lateral Jerk- The lateral Jerk is defined as the change of lateral acceleration with respect to time created by the centrifugal force on the horizontal curve.

- For the safety and comfortable driving, lateral Jerk must be below the predetermined limit values.

Minimum Horizontal Curve Radius based on the Limit Value of Lateral Jerk

- Jerk equation is as follows (Baybura, 2001):

$$
\begin{aligned}
& \mathrm{Z}_{\mathrm{y}}=\frac{\mathrm{bV}}{\sqrt{\mathrm{u}^{2}+\mathrm{b}^{2}}}\left\{3 k_{Y} a_{T}+V^{2} \frac{\mathrm{~d}_{\mathrm{kY}}}{\mathrm{~d}_{\mathrm{l}}} \pm \frac{\mathrm{u} V^{2}}{\mathrm{~b} \sqrt{1+\mathrm{W}^{2}}} \frac{\mathrm{~d}_{\mathrm{kD}}}{\mathrm{~d}_{\mathrm{l}}}+\left(\frac{-\mathrm{k}_{\mathrm{Y}} \mathrm{~V}^{2} \mathrm{u}}{\mathrm{u}^{2}+\mathrm{b}^{2}}-\frac{g}{b}\right.\right. \\
&\left.+\frac{g u^{2}}{b\left(u^{2}+b^{2}\right)} \pm \frac{k_{D} V^{2}}{\mathrm{~b} \sqrt{1+\mathrm{W}^{2}}} \pm \frac{-k_{D} V^{2} u^{2}}{\mathrm{~b} \sqrt{1+\mathrm{W}^{2}}\left(u^{2}+b^{2}\right)}\right) \frac{\mathrm{d}_{\mathrm{u}}}{\mathrm{~d}_{1}} \\
&\left. \pm \frac{-u^{2} V^{2} k_{D} W}{\mathrm{~b}\left(1+\mathrm{W}^{2}\right)^{3 / 2}} \frac{\mathrm{~d}_{\mathrm{W}}}{\mathrm{~d}_{1}} \pm \frac{2 u k_{D} a_{T}}{\mathrm{~b} \sqrt{1+\mathrm{W}^{2}}}\right\}
\end{aligned}
$$

$\mathrm{Z}_{\mathrm{y}}$ : Lateral Jerk (m/s ${ }^{3}$ )
V : Design speed ( $\mathrm{m} / \mathrm{s}$ )
b : Horizontal width of road platform (m)
u : Superelevation (m)
$\mathrm{k}_{\mathrm{D}}$ : Vertical curvature ( $1 / \mathrm{m}$ )
g : Gravitational acceleration $\left(9,81 \mathrm{~m} / \mathrm{s}^{2}\right)$
$\mathrm{a}_{\mathrm{T}}:$ Resultant tangential acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right)$
W : Longitudinal slope
$\mathrm{k}_{\mathrm{Y}}:$ Horizontal curvature ( $1 / \mathrm{m}$ )
1 : Horizontal length of road

Minimum Horizontal Curve Radius based on the Limit Value of Lateral Jerk

- In the equation;
$\checkmark$ The value of $k_{Y}$ represents the curvature of horizontal geometry at any point. $k_{y}$ is equal to the value of $\left(1 / r_{y}\right)$.
$\checkmark$ The derivatives of the superelevation and radius of curvature with respect to distance are zero.
$\checkmark$ The expression of $\sqrt{\mathrm{u}^{2}+\mathrm{b}^{2}}$ (sloping width of road platform) can be equal to the value of horizontal width of road platform (b).

Minimum Horizontal Curve Radius based on the Limit Value of Lateral Jerk

- The equation can be edited for horizontal geometry ( $\mathrm{k}_{\mathrm{D}}: 0$ ).
$\checkmark$ For highways; equation of minimum curve radius based on the lateral Jerk is:

$$
\mathrm{R}_{\min }=\frac{3 \mathrm{~V}_{\max } \mathrm{a}_{\mathrm{T}}}{\mathrm{z}_{\mathrm{y}}}
$$

$\checkmark$ For railways; equation of minimum curve radius based on the lateral Jerk is:

$$
\mathrm{R}_{\min }=\frac{3 \mathrm{~V}_{\max } \sqrt{\mathrm{b}^{2}-\mathrm{u}_{\max }^{2}} \mathrm{a}_{\mathrm{T}}}{\mathrm{~b} \mathrm{Z}}
$$

Minimum Horizontal Curve Radius based on the Limit Value of Lateral Jerk

- Limit values of lateral Jerk in the literature;

For highways;
$\checkmark 0.3-0.9 \mathrm{~m} / \mathrm{s}^{3}$ (AASHTO, 2001)
$\checkmark 0.6 \mathrm{~m} / \mathrm{s}^{3}$ (Schofield; 2001), (Umar; Yayla; 1997)
$\checkmark 0.5 \mathrm{~m} / \mathrm{s}^{3}$ (Manns; 1985)
For railways;
$\checkmark 0.5 \mathrm{~m} / \mathrm{s}^{3}$ (Megyeri; 1993), (Evren; 2002)
$\checkmark 0.4 \mathrm{~m} / \mathrm{s}^{3}$ (Förstberg; 2000)
$\checkmark 0.2 \mathrm{~m} / \mathrm{s}^{3}$ (Esveld; 1989)

## Numerical Application

- Minimum curve radius based on the different limit values were calculated.

| $\underset{(K \mathbf{K} / \mathrm{h})}{\mathrm{V}}$ | $\mathrm{R}_{\text {min }}(\mathrm{m})$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{\mathrm{y}}\left(\mathrm{m} / \mathrm{s}^{3}\right)$ |  |  |  |  |  |  | $\mathrm{a}_{\mathrm{y}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | e (\%) |
|  | $\mathrm{Z}_{\mathrm{y}}: 0.3$ | $\mathrm{Z}_{\mathrm{y}}: \mathbf{0 . 4}$ | $\mathrm{Z}_{\mathrm{y}}: 0.5$ | $\mathrm{Z}_{\mathrm{y}}: \mathbf{0 . 6}$ | $\mathrm{Z}_{\mathrm{y}}: 0.7$ | $\mathrm{Z}_{\mathrm{y}}: 0.8$ | $\mathrm{Z}_{\mathrm{y}}: 0.9$ | $\mathrm{a}_{\mathrm{y}}: 1.47$ | e: 4 |
| 20 | 115 | 85 | 70 | 60 | 50 | 45 | 40 | 15 | 15 |
| 30 | 170 | 125 | 100 | 85 | 75 | 65 | 60 | 35 | 35 |
| 40 | 225 | 170 | 135 | 115 | 100 | 85 | 75 | 55 | 60 |
| 50 | 280 | 210 | 170 | 140 | 120 | 105 | 95 | 85 | 100 |
| 60 | 335 | 250 | 200 | 170 | 145 | 125 | 115 | 125 | 150 |
| 70 | 390 | 300 | 235 | 195 | 170 | 150 | 170 | 170 | 215 |
| 80 | 445 | 335 | 270 | 225 | 195 | 170 | 150 | 220 | 280 |
| 90 | 500 | 375 | 300 | 250 | 215 | 190 | 170 | 280 | 375 |
| 100 | 560 | 420 | 335 | 280 | 240 | 210 | 190 | 345 | 495 |
| 110 | 615 | 460 | 370 | 310 | 265 | 230 | 205 | 415 | 635 |
| 120 | 670 | 500 | 400 | 335 | 290 | 250 | 225 | 495 | 875 |
| 130 | 725 | 725 | 435 | 365 | 310 | 275 | 245 | 580 | 1110 |

- Table shows the minimum curve radius based on the different limit values for highways.


## Numerical Application

- $Z: 0.3 \mathrm{~m} / \mathrm{s}^{3}, \mathrm{~V}: 0-100 \mathrm{~km} / \mathrm{h} \rightarrow R_{J}>R_{e}>R_{a}$
$\mathrm{V}: 100-130 \mathrm{~km} / \mathrm{h} \rightarrow R_{e}>R_{J}>R_{a}$
$\mathrm{Z}: 0.5 \mathrm{~m} / \mathrm{s}^{3}, \mathrm{~V}: 0-70 \mathrm{~km} / \mathrm{h} \rightarrow R_{J}>R_{e}>R_{a}$
$\mathrm{V}: 70-90 \mathrm{~km} / \mathrm{h} \rightarrow R_{e}>R_{J}>R_{a}$
$\mathrm{V}: 100-130 \mathrm{~km} / \mathrm{h} \rightarrow R_{e}>R_{a}>R_{J}$
$\mathrm{Z}: 0.7 \mathrm{~m} / \mathrm{s}^{3}, \mathrm{~V}: 0-50 \mathrm{~km} / \mathrm{h} \rightarrow R_{J}>R_{e}>R_{a}$
V: $50-70 \mathrm{~km} / \mathrm{h} \rightarrow R_{e}>R_{J}>R_{a}$
V: $80-130 \mathrm{~km} / \mathrm{h} \rightarrow R_{e}>R_{a}>R_{J}$
- During the design of horizontal curve, the biggest radius calculated with respect to different criteria must be used for comfortable designing.


## Numerical Application

| $\underset{(K m / h)}{V}$ | Rmin (m) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Z}_{\mathrm{y}}\left(\mathrm{m} / \mathrm{s}^{3}\right)$ |  |  |  | $\mathrm{a}_{\mathrm{y}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathbf{u}_{\text {max }}(\mathrm{m})$ |
|  | $\mathrm{Z}_{\mathrm{r}}: \mathbf{0 . 3}$ | $\mathrm{Z}_{\mathrm{r}}: 0.4$ | $\mathrm{Z}_{\mathrm{r}}: 0.5$ | $\mathrm{Z}_{\mathrm{y}}: 0.6$ | $\mathrm{a}_{\mathrm{y}}$ : 0.65 | $\mathrm{u}_{\text {max }}$ : 0.15 |
| 20 | 110 | 85 | 70 | 55 | 20 | 35 |
| 30 | 165 | 125 | 100 | 85 | 45 | 70 |
| 40 | 225 | 165 | 135 | 110 | 75 | 125 |
| 50 | 280 | 210 | 165 | 140 | 120 | 195 |
| 60 | 335 | 250 | 200 | 165 | 170 | 285 |
| 70 | 390 | 290 | 235 | 195 | 230 | 385 |
| 80 | 445 | 335 | 265 | 225 | 305 | 500 |
| 90 | 500 | 375 | 300 | 250 | 385 | 635 |
| 100 | 555 | 415 | 335 | 280 | 470 | 785 |
| 110 | 610 | 460 | 365 | 305 | 570 | 950 |
| 120 | 665 | 500 | 400 | 335 | 680 | 1130 |
| 130 | 720 | 540 | 435 | 360 | 795 | 1325 |
| 140 | 775 | 580 | 465 | 390 | 925 | 1535 |
| 160 | 885 | 665 | 530 | 445 | 1205 | 2005 |
| 180 | 995 | 750 | 600 | 500 | 1525 | 2535 |
| 200 | 1105 | 830 | 665 | 555 | 1885 | 3135 |
| 220 | 1220 | 915 | 730 | 610 | 2280 | 3790 |
| 250 | 1382 | 1040 | 830 | 690 | 2945 | 4890 |

- Table shows the minimum curve radius based on the different limit values for railways.


## Numerical Application



Minimum horizontal curve radius depends on the lateral Jerk for highways


## conclusion

- In this study as distinct from similar studies, minimum curve radius used as design criterion on the design of horizontal geometry of alignment was calculated to take into consideration lateral Jerk criterion.
- The equations of minimum curve radius based on lateral Jerk were derived from the equations of lateral Jerk in the literature.
- Minimum curve radius calculated according to different values of jerk was compared to minimum curve radius calculated according to other methods.
- The results were interpreted in terms of road safety and comfort. The biggest minimum curve radius calculated with respect to different criterion could be used for the design of comfortable and safety highways and railways.


# Thank you very much for your attention 

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