PPP Application for estimation of precise point coordinates – case study of a reference station in Nigeria

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Abstract

Precise satellite orbits and clocks from the International GPS Service (IGS) and International Reference Frame (ITRF) have been available for years. Their accuracy and latency improves but very slightly in recent years. Now that Selective Availability (SA) is permanently switched off, PPP can satisfy the most demanding Global Navigation Satellite system (GNSS) users observing at high data rates. With dual-frequency pseudorange and carrier-phase observations, stand-alone GNSS users can now consistently achieve, in postprocessing and on a global scale, static and kinematic positioning with cm to dm accuracy. This level of accuracy is possible when GNSS users take advantage of very precise satellite clock estimates available with the orbits, while accounting for all known systematic effects affecting their observations. This work details a post-processing approach that uses undifferenced dual-frequency pseudorange and carrier phase observations along with IGS precise orbit products, for stand-alone precise geodetic point positioning static with centimeter precision. This is possible if one takes advantage of the satellite clock estimates available with the satellite coordinates in the IGS precise orbit products and models systematic effects that cause centimeter variations in the satellite to user range. This paper will describe the approach, summarize the adjustment procedure and specify the earth and space based models that must be implemented to achieve centimeter level positioning in static mode. Results obtained using existing control as case studies are also presented.

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates – case study of a reference station in Nigeria

1.0 Introduction

PPP processing of undifferenced smoothed pseudoranges with fixed precise satellite orbits and clocks has been used by GSD since 1992 [1]. For applications requiring meter level positioning, this approach has satisfied a number of users. These products satisfy GNSS users observing at high data rates in either static or kinematic modes for applications requiring meter precision. For GNSS users seeking to achieve geodetic precision, sophisticated processing software such as GIPSY, BERNESE. and GAMIT[13] are required. By using the IGS precise orbit products and combining the GNSS carrier phase data with reference station observations, geodetic users achieve precise positioning while integrating into the ITRF[1]. Software provided by receiver manufacturers may also be used as long as it allows for the input of station and orbit data. For a number of years, precise point positioning algorithms using undifferenced carrier phase observations have also been available in the GIPSY GNSS analysis software [1]

The AUSPOS Online GPS Processing Service uses International GNSS Service (IGS) products (final, rapid, ultra-rapid depending on availability) to compute precise coordinates in International Terrestrial Reference Frame (ITRF) anywhere on Earth and GDA94 within Australia. The Service is designed to process only dual frequency GNSS data. AUPOS -PPP uses the so-called point positioning approach user do not need to be near an Active control points (ACP), IGS or continuous reference station (CORS). Unlike differential GNSS methods, PPP does not require data from any other GNSS receivers. Instead PPP uses precise apriori values of the GNSS orbits and of their clocks. These values are obtained from the International GNSS Service (IGS) and are usually available, in batches of 24 hours, 17 hours after the end of the GNSS day. The precise orbits and clocks remove a large part of the GNSS errors. In addition, PPP processing must also properly account for several other effects on the position of the GNSS receiver [11].

The approach presented here is an implementation of precise point positioning that was effectively submitted data via internet to AUPOS – PPP center for observation carried out at BM_AMOS at Jebba Dam Nigeria. The result was also compared with the coordinate obtained using Canadian Spatial Reference System Precise Point Positioning (CSRS-PPP) at the same station.

2.0 Precise Point Positioning

It has been shown that code-based point positioning solution could be improved to match the Differential Global positioning systems (DGPS) solution through the use of ionosphere-free, undifferenced pseudorange with precise ephemeris and clock data. To achieve the highest possible point positioning accuracy, both carrier-phase data should be used. In addition, the remaining unmodelled errors, namely tropospheric delay, satellite altitude error, and site displacement effect, must be dealt with. This approach is commonly known as the Precise Point Positioning, or PPP [1]. Symbol ρ is the geometrical range computed as a function of satellite (Xs, Ys, Zs) and station (x, y, z) coordinates according to [1]:

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

$$\rho = (\Delta X^2 + \Delta Y^2 + \Delta Z^2)^{1/2}$$
 (1)

The ionospheric-free combinations of dualfrequency GNSS pseudorange (P) and carrierphase observations (Φ) are related to user position, clock, troposphere and ambiguity parameters according to equations below [1].

$$\ell_{p} = \rho + C(dt - dT) + M ztd + \varepsilon P \tag{2}$$

$$\ell_{\Phi} = \rho + C(dt - dT) + M ztd + N \lambda + \varepsilon \Phi \tag{3}$$

where:

 ℓ_p is the ionosphere-free combination of L1 and L2 pseudoranges (2.54P₁-1.54P₂)

is the ionosphere-free combination of L1 and L2 carrier-phases $(2.54\phi_1 - 1.54 \phi_2)$

dt is the station clock offset from GPS time,

dT is the satellite clock offset from GPS time,

C is the vacuum speed of light,

 λ is the carrier, or carrier-combination, wavelength,

N is the ambiguity of the carrier-phase ionosphere-free combination,

M is function to map tropospheric from slant to zenith,

ztd is the signal tropospheric zenith total delay due to the neutral-atmosphere, wavelength, ionosphere-free combination.

 εP , $\varepsilon \Phi$ are the relevant measurement noise components, including multipath

3.0 Adjustment Model

Given precise estimates of GPS satellite orbits and clocks, equations (2) and (3) reduce to [2]:

$$P = \rho + Cdt + Mztd + \varepsilon P \tag{4a}$$

$$\Phi = \rho + Cdt + Mztd + N\lambda + \varepsilon\Phi \tag{4b}$$

Linearization of observation equations (4a) and (4b) around the a-priori parameters and observations (X,) becomes, in matrix form:

$$A \delta + W - V = 0, \tag{5}$$

where *A* is the design matrix, δ is the vector of corrections to the unknown parameters *X*, $W = f(X^0, \ell)$ is the misclosure vector and *V* is the vector of residuals.

The partial derivatives of the observation equations with respect to X, consisting of four types of parameters:

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

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station position (x,y,z), clock (dt), troposphere zenith path delay (zpd) and (non-integer) carrier-phase ambiguities (N), form the design matrix A[1]:

$$A = \begin{bmatrix} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{x}} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{y}} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{z}} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{dt}} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{zpd}} \frac{\partial f\left(X, \ell_{p}\right)}{\partial X_{N_{(j=1, nsat)}}} \\ \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{x}} \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{y}} \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{z}} \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{dt}} \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{zpd}} \frac{\partial f\left(X, \ell_{\Phi}\right)}{\partial X_{N_{(j=1, nsat)}}} \end{bmatrix} - - - - (6)$$

where

$$\frac{\partial f}{\partial X_x} = \frac{x - X_s}{\rho}, \qquad \frac{\partial f}{\partial X_y} = \frac{y - Y_s}{\rho}, \qquad \frac{\partial f}{\partial X_z} = \frac{z - Z_s}{\rho}$$

$$\frac{\partial f}{\partial X_{dt}} = C, \frac{\partial f}{\partial X_{zpd}} = M, \frac{\partial f}{\partial X_{N_{(i-1,nyd)}}} = 0$$

$$X = \begin{bmatrix} x \\ y \\ z \\ dt \\ zdt \\ N_{(j=1,nsat)} \end{bmatrix}$$

The least squares solution with *a-priori* weighted constraints (p_x) to the parameters is given by [2]:

$$\delta = -(P_{X0} + A^T P_{\ell} A)^{-} A^T P_{\ell} W \tag{7}$$

$$\bar{X} = X^0 + \delta \tag{8}$$

with covariance matrix

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

$$C_{\bar{X}} = P_{\bar{X}}^{-1} = (P_{X^0} + A^T P_{\ell} A)^{-1}$$
(9)

4.0 Adjustment Procedure

The adjustment procedure developed is effectively a sequential filter that adapts to varying user dynamics. The implementation considers the variations in the states of the parameters between observation epochs and uses appropriate stochastic processes to update their variances [4]. The current model involves four types of parameters: station position (x, y, z), receiver clock (dt), troposphere zenith path delay (zpd) and carrier-phase ambiguities (N). The station position may be constant or change over time depending on the user dynamics. These dynamics could vary from tens of meters per second in the case of a land vehicle to a few kilometers per second for a low earth orbiter [6]. The receiver clock will drift according to the quality of its oscillator, e.g. several centimeters/second in the case of an internal quartz clock with frequency stability of about 10-10. Comparatively, the zenith path delay will vary in time by a relatively small amount, in the order of a few centimeters/hour. Finally, the non-integer carrier-phase ambiguities (N) will remain constant as long as the carrier phases are free of cycle-slips, a condition that requires close monitoring. (Note that for double differenced data, (dt) is practically eliminated and the carrier-phase ambiguities (N) become integers) [6].

Using subscript i to denote a specific time epoch, we see that without observations between epochs, initial parameter estimates at epoch i are equal to the ones obtained at epoch i-1[2]: Hence,

$$X_{i}^{0} = X_{i-1}^{-}$$
10)

To propagate the covariance information from epoch i-1 to i, during an interval Δt , it has to be updated to include process noise represented by the covariance matrix $C\epsilon\Delta t$ [7]

Thus,

$$P_{X_i^0} = \left[C_{\bar{X}_{i-1}} + C\varepsilon_{\Delta t}\right]^{-1} \tag{11}$$

4.1 Resolving the integer ambiguities

The process of resolving the integer ambiguities is called 'initialization'[5]. Unlike the double differenced carrier phase, the undifferenced PPP analysis estimates only real numbers of ambiguities, or floated solutions. In fact, PPP ambiguities are reparameterized

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

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ambiguities which are collections of ambiguities of L1 and L2, with carrier phase ionosphere-free scaling factors. Figure 1 is the Ambiguities result from the submitted GPS observation at BM_AMOS. The reparameterized ambiguities (N) are estimated every epoch, as a pure random walk process (Langley, 1999). The dynamic model is in the form:

where w is Gaussian noise.

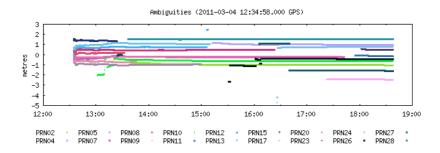


Figure 1 – Ambiguities

Below is the distribution of ITRF stations used as reference points.

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

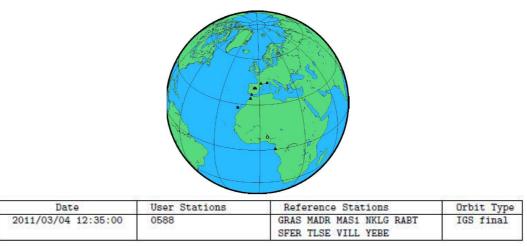


Figure 1- is the distribution of ITRF stations

Table 1- is the Cartesian, ITRF2008

Station	X (m)	Y (m)	Z (m)	ITRF2008 @ 04/03/2011	
0588	6275737.771	525566.758	1007172.224		
GRAS	4581690.817	556114.954	4389360.865	04/03/2011	
MADR	4849202.300	-360328.845	4114913.273	04/03/2011	
MAS1	5439192.200	-1522055.366	2953454.956	04/03/2011	
NKLG	6287385.743	1071574.656	39133.031	04/03/2011	
RABT	5255617.622	-631745.581	3546322.640	04/03/2011	
SFER	5105518.942	-555145.770	3769803.451	04/03/2011	
TLSE	4627851.764	119640.136	4372993.626	04/03/2011	
VILL 4849833.631		-335048.902	4116015.009	04/03/2011	
YEBE	4848724.649	-261632.089	4123094.229	04/03/2011	

Geoid – ellipsoidal separation in this section are computed using a spherical harmonic synthesis of the global EGM2008 geoid. Below is the geodetic, GRS80 Ellipsoid, ITRF2008

Table 2- is the geodetic, GRS80 Ellipsoid, ITRF2008

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

Station		Latitude			Longitude		Ellipsoidal	Derived Above
			(DMS)			(DMS)	Height(m)	Geoid Height(m)
0588	9	08	47.01380	4	47	13.60037	133.710	109.632
GRAS	43	45	17.05928	6	55	14.06935	1319.310	1268.247
MADR	40	25	44.98660	-4	14	58.76756	829.433	776.357
MAS1	27	45	49.47249	-15	37	59.78852	197.156	153.618
NKLG	0	21	14.06747	9	40	19.65388	31.502	21.516
RABT	33	59	53.17671	-6	51	15.43769	90.086	44.738
SFER	36	27	51.64729	-6	12	20.32061	84.154	39.146
TLSE	43	33	38.50419	1	28	51.21096	207.193	157.730
VILL	40	26	36.93927	-3	57	07.12015	647.338	595.388
YEBE	40	31	29.64524	-3	05	19.04935	972.760	920.259

The ambiguity should be constant as long as the receiver is locked to the signal. The ambiguity number changes when the receiver loses lock. Unlike the double differenced carrier phase, the undifferenced PPP analysis estimates always lead to real valued ambiguities, or floated solutions. In fact, PPP ambiguities are reparameterized ambiguities which are collections of ambiguities of L1 and L2, with carrier phase ionosphere-free scaling factors [3]. Figure 2 is the Ambiguities resolution from the submitted GPS observation at station BM_AMOS.

The reparameterized ambiguities (N) are being estimated every epoch, as a pure random walk process. The dynamic model is in the form [13]:

where w is Gaussian noise.

Table 3 – Ambiguities Resolution at BM AMOS

Baseline	Ambiguities Deselved	Deceline Length (lm)
baseline	Ambiguities Resolved	Baseline Length (km)
NKLG - TLSE	41.2 %	4737.4
MAS1 - RABT	52.9 %	1085.3
TLSE - YEBE	81.2 %	506.6
RABT - SFER	82.4 %	279.9
SFER - TLSE	81.2 %	1023.4
TLSE - VILL	82.4 %	567.5
GRAS - TLSE	83.3 %	439.2
MADR - TLSE	76.5 %	588.2
0588 - MAS1	46.7 %	2946.3
AVERAGE	70.0%	1352.6
AVEIMOL	10.0%	1302.0

TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates – case study of a reference station in Nigeria

Solution such as used by AUSPOS, an average ambiguity resolution of 50% or better for the network indicates a reliable solution.

5.0 PPP Solution Convergence

PPP must rely on carrier phase data when centimeters are being sought. It means that integer ambiguities must be resolved (with the help of code data), data errors must be modelled and the instrumental noise level of the data must be as low as possible. There are still other problems such as a multi-path of GNSS signals. At the initial epoch, because of unknown carrier-phase ambiguities, the solution relies entirely on the pseudorange observations and the quality of the position reflects GNSS receiver code resolution and the multipath environment at the tracking station. As time passes and phase observations are added to the solution, the ionospheric free ambiguities and station position components (in static mode) converge to constant values while the troposheric zpd and receiver clock parameters vary as a function of their assigned process noise [14].

5.1 RESULTS OF ADJUSTMENTS OF OBSERVATIONS

The qualities to be obtained after field measurements and adjustment in a GPS network include baseline vector components and their covariance. We present in Table 5 summary of the coordinates obtained using AUPOS and CSRS- PPP online processing results at station BM_AMOS at Jebba Dam. The results show consistent in the horizontal component and with 1.41m in terms of Ellipsoidal height. We cannot conclude yet if the station has positional error till more observations are carried out using CORS in Nigeria as reference point. Table 4 is the standard deviation of all the station used to fix the position of BM_AMOS.

Table 4 - Processing Summary of one of the observation at BM_AMOS

Station	σ East (m)	σ North (m)	σ Up (m)
0588	0.002	0.001	0.006
GRAS	0.001	0.001	0.002
MADR	0.001	0.001	0.002
MAS1	0.001	0.001	0.002
NKLG	0.001	0.001	0.003
RABT	0.001	0.001	0.002
SFER	0.001	0.001	0.002
TLSE	0.001	0.001	0.002
VILL	0.001	0.001	0.002
YEBE	0.001	0.001	0.002

Table 5 – Summary of results obtained using AUPOS and CSRS – ppp

Station coordinate for BM AMOS obtained using AUPOS – PPP in WGS84

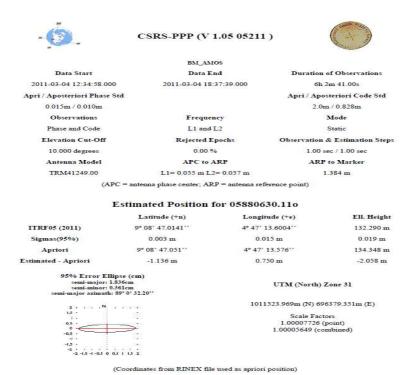
TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

Station	Latitude	Longitude	Ellipsoidal	Derived Above	
	(DMS)	(DMS)	Height(m)	Geoid Height(m)	
0588	9 08 47.01380	4 47 13.60037	133.710	109.632	

Station coordinate for BM_AMOS obtained using CSRS – PPP in WGS84



The existing orthometric height at the station is 108.669m while the derived above Geoid height using AUPOS – PPP was found to be 109.632m.



TS02B - Geodetic Datum II, 5496
Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR
PPP Application for estimation of precise point coordinates – case study of a reference station in Nigeria



Figure 2- BM_AMOS Geodetic Station at Jebba Dam

6.0 Conclusion

This study has evaluated the use of Precise Point Positioning (PPP) in Nigeria by processing data obtained at station BM_AMOS GNSS receivers over a period of time.

The observation equations, estimation technique and correction models used for GNSS PPP using IGS orbit/clock products were described. Results from the post-processing of station dual-frequency pseudorange and carrier phase observations from stationary GPS receivers at BM_AMOS station shows cm positioning precision. The resulting coordinate has been used in the deformation and erosion studies at the dam site.

From the above inferences, it can finally be concluded that there is need for Surveyors in Nigeria to embrace the use of PPP instead of looking for existing Geodetic control to conduct Survey.

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TS02B - Geodetic Datum II, 5496

Prof. Vladimir A.SEREDOVICH, R. EHIGIATOR-IRUGHE and M. O. EHIGIATOR

PPP Application for estimation of precise point coordinates - case study of a reference station in Nigeria

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