Verification of a Two-Stage Valuation Model for Different Types of Real Estates in Local Markets[1]

Anna BARANSKA, Poland

Key words: specifying of market value, covariance matrix of residuals, two-stage valuation

SUMMARY

The paper presents the construction of a two-stage real estate valuation algorithm based on probabilistic models of varying real estate prices in relation to their attributes. The two-stage valuation model aims to make more precise the prediction of a real estate value determined with a functional model which parameters are estimated on the basis of market data.

The first stage of the algorithm includes the estimation of parameters of a multidimensional functional model fitted the best possibly to the local real estate market of a selected type. Estimated model is subject to a versatile statistical verification. If the result is positive, we predict real estate values using the model.

The second stage includes a detailed analysis of variance of model residuals obtained in the first stage. Within this stage, we select a group of real estates most similar to the estimated one, among these, which had been used as input data for the model parameter estimation process at the first stage. From the residuals for selected real estates, a random correction for the prediction of estimated real estate market value received at the first stage is calculated. The addition of this correction makes the prediction of real estate value more precise by taking into consideration the data of the real estates most similar to the estimated one.

The final prediction of the real estate value is given with a full analysis of variance, so it can be presented as point and interval estimation of the result.

The presented algorithm aims to predict real estate market values by a two-stage method with a full analysis of variance at each stage. It shows an increase of prediction accuracy in relation to the value received directly from the estimated model.

In the final part of the work, is presented practical numerical examples of application and a statistical verification of a two-stage valuation model, basing on additive and multiplicative multidimensional models, estimated using databases of real estates in southern Poland.

[1] The task within statutory research in Geomatic Department,
University of Science and Technology AGH, Krakow, Poland

Słowa kluczowe: doprecyzowanie wartości rynkowej, macierz kowariancji odchyłek losowych, dwuetapowa wycena

STRESZCZENIE

W artykule zaprezentowano konstrukcję dwuetapowego algorytmu wyceny nieruchomości, opartego na probabilistycznych modelach zmienności cen nieruchomości względem ich cech charakterystycznych (atrybutów). Dwuetapowy model wyceny ma na celu doprecyzowanie prognozy wartości rynkowej nieruchomości uzyskanej na podstawie modelu funkcyjnego, którego parametry są szacowane na podstawie danych rynkowych.

Pierwszy etap algorytmu obejmuje estymację parametrów wielowymiarowego modelu funkcyjnego, dopasowanego możliwie najlepiej do lokalnego rynku nieruchomości wybranego typu. Wyestymowany model podlega wszechstronnej weryfikacji statystycznej. Po jej pozytywnym wyniku, prognozujemy wartości nieruchomości na podstawie modelu.
Etap drugi obejmuje szczegółową analizę wariancji odchyłek losowych modelu, uzyskanych w etapie pierwszym. W ramach tego etapu, przy pomocy różnych algorytmów jakościowych, wybierana jest grupa nieruchomości najbardziej podobnych do wycenianej, spośród tych, które posłużyły za dane wejściowe do procesu estymacji parametrów modelu w etapie pierwszym. Z odchyłek losowych dla wybranych nieruchomości jest obliczana poprawka losowa, o którą zostanie poprawiona prognoza rynkowej wartości nieruchomości wycenianej, uzyskana z etapu pierwszego. Poprzez dodanie w/w poprawki następuje doprecyzowanie predykcji wartości nieruchomości z uwagi na uwzględnienie informacji o nieruchomościach najbardziej podobnych do wycenianej.

Ostateczna prognoza wartości rynkowej nieruchomości podana jest z pełną analizą wariancji, a zatem może być przedstawiona w formie punktowego i przedziałowego oszacowania wyniku.

Zaprezentowany algorytm ma na celu dochodzenie do prognozy wartości rynkowej nieruchomości, na drodze dwustopniowego jej urealniania, wraz z pełną analizą wariancji na każdym etapie. Pokazuje on zwiększenie dokładności ostatecznej prognozy w stosunku do wartości uzyskanej wprost z wyestymowanego modelu.

W końcowej części pracy zostały przedstawione praktyczne przykłady liczbowe zastosowania i statystycznej weryfikacji dwuetapowego modelu wyceny, oparte na addytywnych oraz multiplikatywnych wielowymiarowych modelach, wyestymowanych na podstawie trzech baz danych o nieruchomościach z terenów Polski południowej.

Verification of a Two-Stage Valuation Model for Different Types of Real Estates in Local Markets[2]

Anna BARANSKA, Poland

1. INTRODUCTION

The process of determining market or cadastral value of a real estate includes nowadays the application of more or less complicated algorithms usually basing on a relatively small number of market data. Even when the number of market data is important and allows to estimate the parameters of multidimensional functional models, achieved in such a way model value of the valuated real estate constitutes at the same time the prediction of its value. Considering a large diversification of input data, such prediction is charged with a considerable standard deviation, which reduces its reliability.

Another method used in practice, to determine the market value is based on a more coherent market database, which implies that the number of data is very considerably restrained. In such a situation, there is no possibility to estimate the accuracy of the final prediction. Even when we succeed to get statistically more accurate estimation of the predicted value, we still have doubts if the result is representative in relation to the whole analysed market of real estates.

Consequently, the proposed two-stage model for determining a real estate value seems to be an optimal solution. It satisfies the condition that input data used at the first stage are representative in consideration of their review character and large number and, at the same time, allows at the second stage, to precise a real estate model value by correcting it on the basis of the parameters obtained from a coherent, strongly confined database having features very close to the valuation object.

2. TESTED VALUATION MODELS

2.1. Additive model

$$w = a_0 + \sum_{k=1}^{m} g_k\left(X_k\right) \qquad (1)$$

where:

- w – unit price or value of real estate,
- X_k – value of attribute k for real estate,
- g_k – function of real estate price - attribute k relation,
- a_0 – constant in the model (unit value of a real estate, for zero of all attributes).

---

[2] The task within statutory research in Geomatic Department, University of Science and Technology AGH, Krakow, Poland

First of all, we will consider a special case of a system of equations in form (1), which is a linear multiple regression (linear non-only by parameters, but also by independent variables):

$$w = a_0 + \sum_{k=1}^{m} X_k \cdot a_k \qquad (2)$$

The expression (2) may be written as a matrix:

$$[W] = [X] \cdot [a] \qquad (3)$$

where:

$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$ – vector of dependent random variable (real estate values),

$X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1m} \\ 1 & x_{21} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nm} \end{bmatrix}$ – matrix containing ones and independent variables (attributes),

$a = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix}$ – vector of multiple linear regression parameters.

In the general structure of an additive model (1), we will find polynomials of different degrees as functions g. Thus, developing the model (1), we have:

$$w = a_0 + \sum_{k=1}^{m} (a_{k_1} \cdot X_k + a_{k_2} \cdot X_k^2 + ... + a_{k_{n_k}} \cdot X_k^{n_k}) \qquad (4)$$

where:

m – number of attributes considered in the model,
n_k – degree of polynomial for k-th attribute.

At polynomial forms of the function g, the whole model maintains the linearity in relation to the parameters. Therefore, the estimation of the model parameters may be done the same way as in the case of multiple regression. The matrix X takes on the form:

$$X = \begin{bmatrix} 1 & x_{11} & x_{11}^2 & \cdots & x_{11}^{n_1} & x_{12} & x_{12}^2 & \cdots & x_{12}^{n_2} & \cdots & x_{1m} & x_{1m}^2 & \cdots & x_{1m}^{n_m} \\ 1 & x_{21} & x_{21}^2 & \cdots & x_{21}^{n_1} & x_{22} & x_{22}^2 & \cdots & x_{22}^{n_2} & \cdots & x_{2m} & x_{2m}^2 & \cdots & x_{2m}^{n_m} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n1}^2 & \cdots & x_{n1}^{n_1} & x_{2n} & x_{2n}^2 & \cdots & x_{2n}^{n_2} & \cdots & x_{nm} & x_{nm}^2 & \cdots & x_{nm}^{n_m} \end{bmatrix} \qquad (5)$$

In table 2, in section 5.2, are presented results of estimation for the following model:

$$w = a_0 + a_{1_1} \cdot X_1 + a_{2_2} \cdot X_2^2 + a_{3_1} \cdot X_3 + a_{4_1} \cdot X_4 + a_{5_1} \cdot X_5 + a_{6_1} \cdot X_6 + a_{6_2} \cdot X_6^2 + a_{7_1} \cdot X_7 \qquad (5a)$$

where X_i are describing the following attributes of agricultural real estates:
X_1 – data of transaction,
X_2 – the shape of a lot,
X_3 – the lot development,
X_4 – designation of a lot,
X_5 – topography of a lot,
X_6 – law restrictions concerning a lot,
X_7 – lot area.

2.2. Multiplicative model

$$w = a_0 \cdot a_1^{x_1} \cdot a_2^{x_2} \cdot \ldots \cdot a_m^{x_m} \qquad (6)$$

where:
w – unit price or value of a real estate,
$x_1, x_2, \ldots, x_m$ – attributes of real estates,
a_j – estimated model parameters,
a_0 – constant in the model (unit value of a real estate, for zero of all attributes).

Estimation of parameters of both valuation model forms can be done by the least squares method. A detailed estimation algorithm provided with the accuracy analysis and the description of statistical methods to verify estimated model is presented in ref. [5].

3. ISOLATING THE SYSTEMATIC FACTOR AND THE RANDOM COMPONENT OF THE MODEL

Estimated valuation model leads to isolating its systematic factor from unit prices of real estates. They are predicted values of real estate prices W determined using the model. Differences between real market prices assembled in database C and model values W are model residuals:

$$[\delta] = [C] - [W] \qquad (7)$$

where:
$[\delta]$ – valuation model residuals,
$[C]$ – vector of real estate prices in data base,
$[W]$ – vector of real estate model prices.

To each systematic factor corresponds a set of model random residuals δ having the inaccuracy characteristics contained in its covariance matrix $Cov[\delta]$:

$$[\delta] = [C] - [X] \cdot [X]^{+} \cdot [C] = \left[I - X \cdot X^{+} \right] \cdot [C] \qquad (8)$$

$$Cov[\delta] = \sigma_0^2 \cdot \left[I - X \cdot \left(X^T \cdot X \right)^{-1} \cdot X^T \right] \qquad (9)$$

where:
X^+ – pseudo-inverse of matrix X,
σ_0^2 – model residual variance.
From the residuals for selected real estates and from their covariance matrix, a random correction for the prediction of estimated real estate market value is calculated.

4. PREDICTION OF REAL ESTATE MARKET VALUE

4.1. Point estimation of real estate model value

Basing on the parameters of a selected model and on the attributes of an estimated real estate, we determine its model value w_M with full accuracy analysis. The analysed real estate value determined in such a way, being a prediction of estimated model, can be its probable market value. In the case of an additive model in multiple regression form, the prediction of value of real estate selected from a given market, is performed according to the following formula:

$$w_M = [1 \quad x_1 \quad x_2 \quad \ldots \quad x_m] \cdot \hat{a} \tag{10}$$

where:
$[1 \quad x_1 \quad x_2 \quad \ldots \quad x_m]$ – vector of values of estimated real estate attributes,
$\hat{a}$ – vector of estimated model parameters.

The accuracy of such a prediction is evaluated by its variance expressed by formula:

$$\sigma^2(w_M) = [1 \quad x_1 \quad x_2 \quad \ldots \quad x_m] \cdot Cov(\hat{a}) \cdot [1 \quad x_1 \quad x_2 \quad \ldots \quad x_m]^T \tag{11}$$

where:
$Cov(\hat{a})$ – covariance matrix of vector model parameters.

4.2. Selection of real estates most similar

Applying a selected qualitative method, from the database used to estimate model parameters, we choose the group of k real estates most similar to the estimated one. Examples of qualitative methods applicable to this task are presented in ref. [4].

4.3. Correction (making real) of real estate model value

From the vector random component we isolate residuals δ_{i_w}, corresponding to selected most similar real estates, and from the residuals covariance matrix – a sub-matrix $Cov[\delta_w]$ having $(k \times k)$ dimensions and containing elements that correspond to the isolated residuals.

From the systematic model we have determined the real estate model market value w_M (10) with its standard deviation $\sigma(w_M)$ (11). Then, from the random model we estimate the market value of the real estate random component:

$$w_L = \left[\underline{1} \cdot P \cdot \underline{1}^T\right]^{-1} \cdot [\underline{1} \cdot P] \cdot [\delta_w] \tag{12}$$

where:

$\underline{1} = [1 \quad 1 \quad \ldots \quad 1]$ – vector containing ones of dimensions $(1 \times k)$

with the weight matrix being covariance matrix inverse:

$$P = Cov^{-1}[\delta_w] \qquad (13)$$

The standard deviation of the random component $\sigma(w_L)$ is calculated according to the formula:

$$\sigma^2(w_L) = \sigma^2_{0_w} \cdot \left[\underline{1} \cdot P \cdot \underline{1}^T\right]^{-1} \qquad (14)$$

where residual variance $\sigma^2_{0_w}$ is calculated for the group of k selected real estates by formula:

$$\hat{\sigma}^2_{0_w} = \frac{\delta_w^T \cdot P \cdot \delta_w - w_L \cdot \underline{1}^T \cdot P \cdot \delta_w}{k-1} \qquad (15)$$

The final, corrected market value of the estimated real estate is calculated as follows:

$$w_{M+L} = w_M + w_L \qquad (16)$$

The addition of correction w_L to model value w_M makes the prediction of real estate value more precise by taking into consideration the data of the real estates most similar to the estimated one.

Between variances of prediction of a real estate market value, obtained from the model w_M as well as the prediction corrected with the random factor w_{M+L}, the following relation occurs:

$$V(w_M) = V(w_{M+L}) + V(w_L) \qquad (17)$$

Then, a standard deviation of the final market value $\sigma(w_{M+L})$ of a real estate can be calculated by formula:

$$\sigma^2(w_{M+L}) = \sigma^2(w_M) - \sigma^2(w_L) \qquad (18)$$

From expression (18) we obtain the accuracy assessment of final prediction of real estate

value $\sigma(w_{M+L})$.

5. PRACTICAL IMPLEMENTATION OF TWO-STAGE MODEL

Presented above proceeding algorithm was verified on many different local markets for different types of real estates. Every time it brought about improving the market value of the estimated real estate predicted by model, reducing its standard deviation and specifying its value by adding a correction.

In the tables below, are presented some of model estimation results in additive (linear in consideration of independent variables and parameters or in relation to the parameters only) and multiplicative form. We present the results of valuating a dwelling and an allotment in southeast Poland. Representative databases of 68÷100 real estates, gathered on different local markets, constituted preliminary data. Acquired information on transactions concern tree towns in Poland, diversified in respect of factors shaping the prices of real estates. Different valuation models – additive or multiplicative – have been used in particular cases.

5.1. Valuation of a dwelling real estate

Table 1. Results of additive and multiplicative models estimation

	additive linear model		multiplicative model	
	Wieliczka town	Rzeszów town	Wieliczka town	Rzeszów town
n	92	98	93	100
u	11	13	10	13
σ_0	207,78	227,12	0,077	0,101
V	0,08	0,09	0,07	0,09
R^2	0,89	0,75	0,91	0,73
s	0,60	0,75	0,56	0,67
a_0	**752,176**	**2026,47**	**7,053**	**7,524**
a_1	**0,58**	**-0,34**	**0,61**	**-0,27**
a_2	**0,30**	**0,33**	-0,05	**0,35**
a_3	-0,08	-0,12	**-0,11**	-0,10
a_4	-0,29	-0,10	0,00	-0,12
a_5	**0,66**	**0,12**	0,00	**0,13**
a_6	**-0,30**	**-0,49**	**0,22**	**-0,51**
a_7	0,08	0,10	0,00	0,13
a_8	**0,21**	**0,59**	**0,12**	**0,59**
a_9	**0,24**	**0,56**	**0,06**	**0,53**
a_{10}	0,07	**0,36**		**0,34**
a_{11}		**-0,27**		**-0,26**
a_{12}		**-0,17**		-0,12

where:

n – number of real estates in a database,
u – number of model estimated parameters,
σ_0 – estimation standard error,
V – coefficient of variation,
R^2 – coefficient of determination,
s – fraction of model parameters statistically significant (bold type) in their total number,
a_i – model parameters.

We can see that the additive model in linear form is better for forecasting the market value in both towns, but both types of models are suitable enough to predict the market value of a dwelling in Wieliczka town and in Rzeszów town. However the fraction of model parameters statistically significant is not very high in every case.

Final valuation results for a dwelling in Wieliczka:

- model in additive form:

Model value of a real estate: 2919,64 [zł/m^2] = 799,90 [EUR/m^2]
Standard deviation of the model value: 91,32 [zł/m^2] = 25,02 [EUR/m^2]

Random value:	- 96,89 [zł/m^2]	=	- 26,54 [EUR/m^2]
Standard deviation of the random value:	70,66 [zł/m^2]	=	19,36 [EUR/m^2]
Final prediction of the market value:	2822,75 [zł/m^2]	=	773,36 [EUR/m^2]
Standard deviation of the final prediction:	57,85 [zł/m^2]	=	15,85 [EUR/m^2]

- model in multiplicative form:

Model value of a real estate:	2779,97 [zł/m^2]	=	761,64 [EUR/m^2]
Standard deviation of the model value:	78,45 [zł/m^2]	=	21,49 [EUR/m^2]
Random value:	- 76,40 [zł/m^2]	=	- 20,93 [EUR/m^2]
Standard deviation of the random value:	42,40 [zł/m^2]	=	11,62 [EUR/m^2]
Final prediction of the market value:	2703,57 [zł/m^2]	=	740,70 [EUR/m^2]
Standard deviation of the final prediction:	66,01 [zł/m^2]	=	18,08 [EUR/m^2]

Final valuation results for a dwelling in Rzeszów:

- model in additive form:

Model value of a real estate:	1909,06 [zł/m^2]	=	523,03 [EUR/m^2]
Standard deviation of the model value:	113,88 [zł/m^2]	=	31,20 [EUR/m^2]
Random value:	- 115,31 [zł/m^2]	=	- 31,59 [EUR/m^2]
Standard deviation of the random value:	61,78 [zł/m^2]	=	16,93 [EUR/m^2]
Final prediction of the market value:	1793,75 [zł/m^2]	=	491,44 [EUR/m^2]
Standard deviation of the final prediction:	95,66 [zł/m^2]	=	26,21 [EUR/m^2]

- model in multiplicative form:

Model value of a real estate:	1851,94 [zł/m^2]	=	507,38 [EUR/m^2]
Standard deviation of the model value:	115,11 [zł/m^2]	=	31,54 [EUR/m^2]
Random value:	- 112,73 [zł/m^2]	=	- 30,88 [EUR/m^2]
Standard deviation of the random value:	53,92 [zł/m^2]	=	14,77 [EUR/m^2]
Final prediction of the market value:	1739,21 [zł/m^2]	=	476,50 [EUR/m^2]
Standard deviation of the final prediction:	101,70 [zł/m^2]	=	27,86 [EUR/m^2]

where:

w_M – Model value of a real estate, calculated according to the formula (10),

$\sigma(w_M)$ – Standard deviation of the model value (11),

w_L – Random value (12),

$\sigma(w_L)$ – Standard deviation of the random value (14),

w_{M+L} – Final prediction of the market value (16),

$\sigma(w_{M+L})$ – Standard deviation of the final prediction (18).

5.2. Valuation of an agricultural real estate

Table 2. Results of linear and non-linear additive models estimation

	additive linear model		additive non-linear model
	Zielonki town		Zielonki town
n	68	n	68
u	6	u	9
σ_0	22,77	σ_0	21,34
R^2	0,61	R^2	0,67
s	1,00	s	1,00
a_0	**20,4937**	a_0	**32,94**
a_1	**-2,24**	a_{11}	**-2,39**
a_2	**11,01**	a_{22}	**2,19**
a_3	**13,67**	a_{31}	**14,86**
a_4	**8,69**	a_{41}	**-11,49**
a_5	**-28,88**	a_{51}	**9,36**
		a_{61}	**-30,77**
		a_{62}	**23,19**
		a_{71}	**-39,65**

where:

n – number of real estates in a database,
u – number of model estimated parameters,
σ_0 – estimation standard error,
R^2 – coefficient of determination,
s – fraction of model parameters statistically significant (bold type) in their total number,
a_i – model parameters.

In this case we can see that both types of models are suitable to predict the market value of an agricultural parcel in Zielonki commune, however the non-linear model is slightly better. It's worth noting that all of model parameters are statistically significant.

Final valuation results for an agricultural property in Zielonki:

- model in additive linear form (multiple regression):

Model value of a real estate:	75,56 [zł/m^2]	=	20,70 [EUR/m^2]
Standard deviation of the model value:	7,81 [zł/m^2]	=	2,14 [EUR/m^2]
Random value:	0,00 [zł/m^2]	=	0,00 [EUR/m^2]
Standard deviation of the random value:	3,91 [zł/m^2]	=	1,07 [EUR/m^2]
Final prediction of the market value:	75,56 [zł/m^2]	=	20,70 [EUR/m^2]
Standard deviation of the final prediction:	6,76 [zł/m^2]	=	1,85 [EUR/m^2]

- model in additive non-linear form (5a):

Model value of a real estate:	74,73 [zł/m^2]	=	20,47 [EUR/m^2]
Standard deviation of the model value:	8,03 [zł/m^2]	=	2,20 [EUR/m^2]
Random value:	5,44 [zł/m^2]	=	1,49 [EUR/m^2]
Standard deviation of the random value:	4,87 [zł/m^2]	=	1,33 [EUR/m^2]
Final prediction of the market value:	80,17 [zł/m^2]	=	21,96 [EUR/m^2]
Standard deviation of the final prediction:	6,38 [zł/m^2]	=	1,75 [EUR/m^2]

where:

w_M – Model value of a real estate, calculated according to the equation (10),

$\sigma(w_M)$ – Standard deviation of the model value (11),

w_L – Random value (12),

$\sigma(w_L)$ – Standard deviation of the random value (14),

w_{M+L} – Final prediction of the market value (16),

$\sigma(w_{M+L})$ – Standard deviation of the final prediction (18).

On the basis of presented examples, we can see that in the case of all valuations, the standard deviation of the final prediction of the real estate value constitutes 2-10% of its altitude, being always minimum over a dozen percent less then the standard deviation of real estate predicted value got directly from the model. Then, the results achieved are considerably more precise than those obtained using only a function model.

6. SUMMARY

Presented two-stage valuation model aims to make more precise the real estate value determined using a valuation model well fitted to the local market and statistically verified. The advantage of the algorithm is the selection in a database of similar real estates, used to estimate parameters of a multidimensional function model, a group of real estates most similar to the valuated object and to employ them in "correcting" a model value of a real estate.

REFERENCES

[1] Adamczewski Z.: "Elementy modelowania matematycznego w wycenie nieruchomości Podejście porównawcze", Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa, 2006
[2] Baranska A.: 2004 "Criteria of database quality appraisement and choice stochastic models in prediction of real estate market value", FIG Working Week 2004 and 27th General Assembly in Athens, Greece, 22 – 27 May 2004
[3] Baranska A.: 2006 „Estimation of parameters of multiplicative exponential function model for real estate market value prediction", XXIII FIG Congress "Shaping the Change", Munich, Germany, 8-13 October 2006
[4] Barańska A.: "Quality and quantity methods for estimating real estates", XX Jesienna Szkoła Geodezji im. Jacka Rejmana, Polanica Zdrój, Poland, 16-17 September 2007
[5] Baranska A.: 2007 "Statistical verification of real estate estimation models", FIG Working Week 2007 "Strategic Integration of Surveying Services", Hong Kong SAR, China, 13-17 May 2007
[6] Baranska A.: "Two-stage model for estimating real estates", The Polish Real Estate Scientific Society press, v. 15, no. 3-4, Olsztyn, 2007
[7] Baranska A., Luczak A.: "Comparing the results of function model estimation for the prediction of real estate market values in additive and multiplicative form", AGH University of science and technology press, Geomatics and Environmental Engineering, v.1, no.3, Cracow 2007
[8] Czaja J.: „Metody szacowania wartości rynkowej i katastralnej nieruchomości", Kraków, 2001
[9] Czaja J., Preweda E.: „Analiza statystyczna zmiennej losowej wielowymiarowej w aspekcie korelacji i predykcji", UWND AGH, Geodezja, t.2, Kraków 2000
[10] Radhakrishna Rao C.: "Modele liniowe statystyki matematycznej", PWN, Warszawa, 1982

BIOGRAPHICAL NOTES

***Name*: Anna Baranska**
Employment: AGH University of Science and Technology in Krakow Poland (from 01.11.2003)
Degree: Philosophy Doctor of Geodetic Sciences (AGH Krakow, 12.06.2003)
Study: Doctor Study at the Faculty of Mining Surveying and Environment Engineering (AGH Krakow, 10.1998 – 06.2003)
Degree: Master of Geodetic Science, Specialization: Real Estate Administration (AGH Krakow, 23.06.2000)
Study: Master Study at the Faculty of Mining Surveying and Environment Engineering (AGH Krakow, 10.1995 – 06.2000)
Degree: Master of Mathematics, Specialization: Applied Mathematics (Jagiellonian University Krakow, 24.06.1996)

Study: Master Study at the Faculty of Mathematics and Physics Faculty (Jagiellonian University Krakow, 10.1991 – 06.1996)

CONTACTS

PhD Anna Barańska
University of Science and Technology
Faculty of Mining Surveying and Environmental Engineering
Geomatics Department
al. A. Mickiewicza 30
30 – 059 Krakow
POLAND
Tel. + 48 12 617 44 80
Fax + 48 12 617 45 88
E-mail: abaran@agh.edu.pl