RTK GPS Based Sea Piling Engineering: Mathematical Model and Its Application

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Key words: RTK GPS, Sea Piling Engineering, Coordinate Transformation.

SUMMARY

Piling engineering is the first step in the construction of bridges and offshore platforms. However, limitations in the real-time determination of the positions of piling pickets exist. For instance, requirement of line of sight, lack of real-time positioning capability and fragile to the hostile observation environment are typical problems for the traditional surveying techniques.

The use of real-time kinematic Global Position Ssystem (RTK GPS) can integrate the field measurements with the real-time computation into a uniform positioning system, which can significantly improve the productivity and efficiency of the whole system. In this paper, the authors propose a real-time piling positioning system for the determination of the pickets. Relevant algorithms and models are presented. As a case study, results of the developed system for a piling project at a biggest cross-sea bridge, Donghai Bridge in Shanghai of China are introduced.

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1. INTRODUCTION

Total stations and theodolites are widely used in the piling engineering. A total station set up on the bank of sea is employed to determine the picket position in the water. There are apparent drawbacks for this approach. For instance, the requirement of a line of sight proves to be very difficult when the construction site is far away from the shore. Also real-time determination of piling positions cannot be achieved due to the communication problems. The use of RTK GPS can overcome the above drawbacks of traditional techniques and dramatically improve the positioning efficiency and accuracy, and significantly reduce the construction cost.

In RTK GPS based piling engineering, one key issue is to calculate the coordinates of the picket centre in real-time. For achieving this target, instant coordinates in WGS84 need to be transformed to the coordinates in an engineering coordinate system. In this paper, algorithms and mathematical models for coordinate transformation and parameter determination are introduced. As a case study, detailed procedure in using proposed system for a large cross-sea piling engineering in Shanghai, China is presented. It demonstrates the designed system is robust and feasible for any sea piling project.

2. ESTABLISHMENT OF A THREE DIMENSIONAL SHIP FIXED COORDINATE SYSTEM (SFCS)

2.1 Positioning Modes and Instrumentation

Theodolites and total stations are widely used in traditional piling enineering to determine the position of the picket. However, GPS antenna cannot be directly installed on the picket when RTK GPS technology is applied. The only choice is to install the GPS antennas on the ship deck. Generally, three GPS units are used in order to calculate the picket positions and the altitude of the ship in real-time, whilst two electronic distance measurement (EDM) instruments are used to achieve an even higher positioning accuracy, taking the relative movement between the ship and picket into consideration. In the proposed system, 2D tiltmeter is used to estimate the ship altitude. Data communication between the control centre and the instruments is considered in the system design. Detail about the instrumene layout can be seen from Fig. 1.

2.2 Definition of a 3D SFCS

Select a horizontal plane as the height datum when the ship is relatively level. The ordinate axis X of the ship is defined in the height datum plane and coincided with the physical axis of the ship structure. The local abscissa axis Y is perpendicular to the ordinate axis. The locations of the GPS antennas shall be firstly determined, and the projections of two GPS antennas on the ship height datum shall be symmetric while the ship is level. The center O is the intersetion point of the connecting line of the two projections with *X* axis. Point O is then defined as the original point of the 3D Ship Fixed Coordinate System. The direction from point O to the picket center is defined as the positive direction of *X* axis. Axis *Z* is upright and completes a left hand coordinate system (Fig. 2).

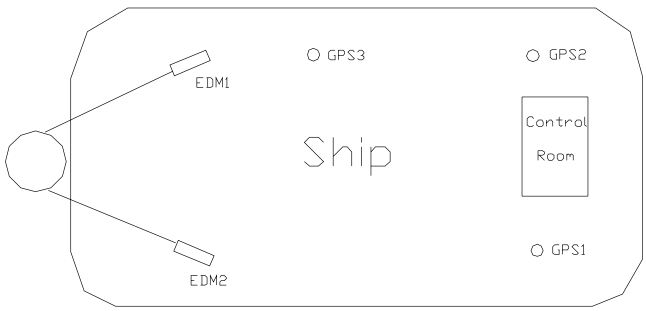

Fig. 1: Instrument Layout on the Ship Deck

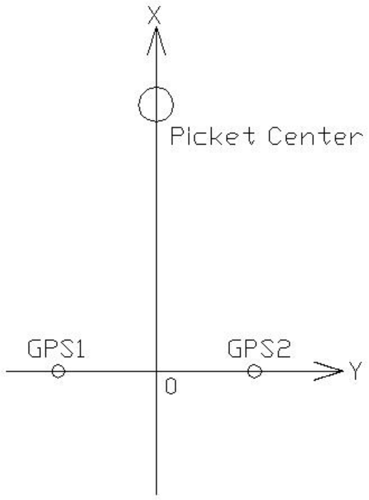

Fig. 2: Establishment of a Ship Fixed Coordinate System

3. COORDINATE SYSTEMS AND THEIR TRANSFORMATION

3.1 Coordinate Systems and Their Transformation

In Fig. 3, O–XYZ is 3D SFCS. Suppose the vertical inclination and cross inclination angles are α and β, respectively. Firstly rotate β along X axis clockwise and get the coordinate system O - XY″Z′, and then rotate an α angle along the Y axis anticlockwise to get the coordinate system O-X″Y″Z″, which defines an instantaneous ship level coordinate system (SLCS). Through translation and rotation the coordinates in SLCS can be transormed into an engineering coordinate system.

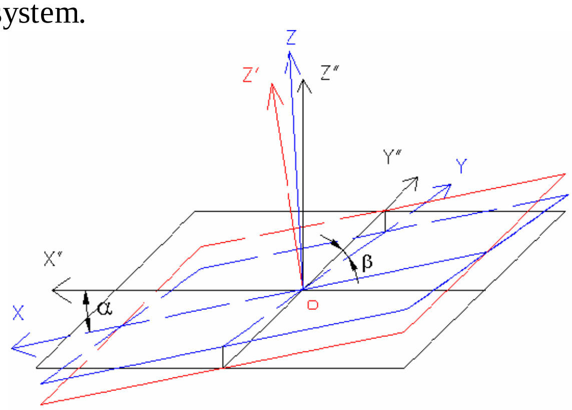

Fig. 3: The transformation between 3D SFCS and instantaneous SLCS

The rotation matrixes between two three-dimension coordinate systems can be expressed as:

$$R_x(\alpha)=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{pmatrix} \qquad (1)$$

$$R_y(\beta)=\begin{pmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{pmatrix} \qquad (2)$$

$$R_z(\gamma)=\begin{pmatrix}\cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{pmatrix} \qquad (3)$$

The rotation angle is determined based on the positive direction of each axis. Anticlockwise rotation angle is defined as positive while the clockwise rotation angel is defined as negative.

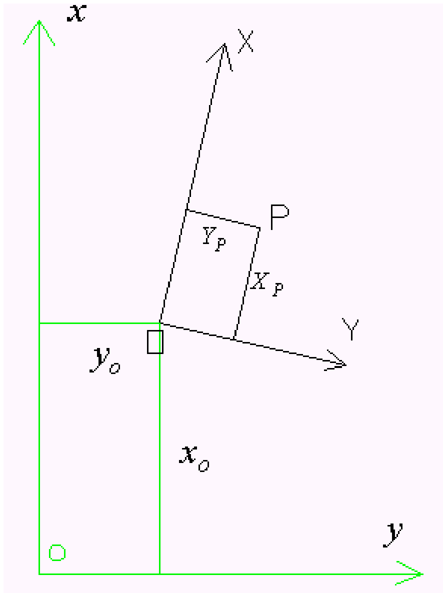

Fig. 4: Transformation between instantaneous SLCS and engineering coordinate system

The transformation matrix from 3D SFCS O-XYZ to the instantaneous SLCS O-X″Y″Z″ is:

$$R = R_X(-\beta)\cdot R_Y(\alpha) \qquad (4)$$

In Fig. 4, a conformal transformation model is usually used to transform coordinates between engineering coordinate system *xoy* and the instantaneous ship level coordinate system XOY. Using translation, rotation, and scale factor the realtionship between two coordinate system can be established and expressed as:

$$\begin{pmatrix}x_P\\ y_P\end{pmatrix}=\begin{pmatrix}X_0\\ Y_0\end{pmatrix}+k\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}*\begin{pmatrix}X_P\\ Y_P\end{pmatrix} \qquad (5)$$

Where α is the clockwise rotation angle from X axis to the x axis. X_0 and Y_0 is the translations, and k is the scale factor.

When more than two ommon points exist, X_0□Y_0□α、k can be estimated using the least squares. Suppose c=$k\cos\alpha$, d=-$k\sin\alpha$, then for ith common point

$$\begin{aligned}x_i &= X_0 + X_i c + Y_i d\\ y_i &= Y_0 + Y_i c - X_i d\end{aligned} \qquad (6)$$

The error equation can be expressed as Eq. 7 when there are r common points

$$x = AX + n \qquad (7)$$

where

$$x=\begin{bmatrix}x_1\\y_1\\x_2\\y_2\\\vdots\\x_r\\y_r\end{bmatrix},\ A=\begin{bmatrix}1&0&X_1&Y_1\\0&1&Y_1&-X_1\\1&0&X_2&Y_2\\0&1&Y_2&-X_2\\\vdots&\vdots&\vdots&\vdots\\1&0&X_r&Y_r\\0&1&Y_r&-X_r\end{bmatrix},\quad n=\begin{bmatrix}x_1-X_1\\y_1-Y_1\\x_2-X_2\\y_2-Y_2\\\vdots\\x_r-X_r\\y_r-Y_r\end{bmatrix},\quad X=\begin{bmatrix}X_0\\Y_0\\c\\d\end{bmatrix}$$

From $X=(A^TPA)^{-1}A^TPl$, X_0, Y_0, c, d can then be determined.

3.2 Determination of Ship's Inclination Angle Using GPS Measurements

If a tiltmeter is used in the positioning system, the ship's inclination angles can then be determined by such an instrument. However, the ship's inclinations can be also determined from the GPS observations. Since three GPS receivers can record the heights in real-time, the inclination angles can be calculated based on the differences of the heights, and also from the height changes in SFCS.

Suppose GPS1, GPS2, GPS3 have their coordinates in SFCS as（X1，Y1，H1），（X2，Y2，H2），and（X3，Y3，H3）. h1，h2，and h3 are the real-time height observations by GPS1、GPS2、GPS3.

When the ship inclines, its vertical and cross inclinations are α and β, repectively. The differences in cooridnates are

X13 **=** X1 - X3，Y13 **=** Y1 - Y3

X23 **=** X2 - X3，Y23 **=** Y2 - Y3, (8)

and the height change between GPS1 and GPS3 is $X13\times\alpha+Y13\times\beta$

Based on the observed heights, the height change between GPS1 and GPS3 can be calculated as (h1-h3)-(H1-H3)

The relationship can then be expressed as $X13\times\alpha+Y13\times\beta=$(h1-h3)-(H1-H3)

Similar equation can be developed $X23\times\alpha+Y23\times\beta=$(h2-h3)-(H2-H3)

From these two equations, inclinations can be estimated as:

$$\alpha=\frac{[(h2-h3)-(H2-H3)]*Y13-[(h1-h3)-(H1-H3)]*Y23}{X23*Y13-X13*Y23} \quad (9)$$

$$\beta = \frac{[(h2-h3)-(H2-H3)]*X13-[(h1-h3)-(H1-H3)]*X23}{X13*Y23-X23*Y13} \quad (10)$$

4. 3D COORDINATE COMPUTATION OF THE PICKET CENTRE

The coordinates of the picket center on the design plane can be computed differently because of using different positioning models. The positioning models here can be classified into two approaches: coarse positioning model and precise positioning model. The coarse positioning model features that the picket is regarded as immobile relative to the ship, and only three GPS are used to determine the positions. Whilst precise positioning model is used the relative movement of picket to the ship is considered. In the latter case, two EDMs are included to obtained more precise positions.

4.1 Coarse Positioning Model for Picket Center Coordinates

Since the picket is assumed as immobile in this positioning mode, the coordinates of the picket centre can be directly calculated.

In Fig. 5, XOZ belongs to SFCS, and the centre of the circumgyrate bracket has coordinates as $(X_R, 0, H_R)$. Assume the vertical inclination angle of the ship be α and the inclinaion of the picket itself γ. The distance between the circumgyrate bracket centre and the picket holder is $(X_Z - X_R)$. X_Z is the coordinate of the picket center when the picket is upright. X_Z, X_R, and H_R can be determined in advance. The X coordinate of the intersection point between picket central line and the OX axis in SFCS can be estimated by Eq. 11.

$$X = X_R + \frac{(X_Z - X_R)}{\sin(\gamma - \alpha)} + H_R \tan(\gamma - \alpha) \quad (11)$$

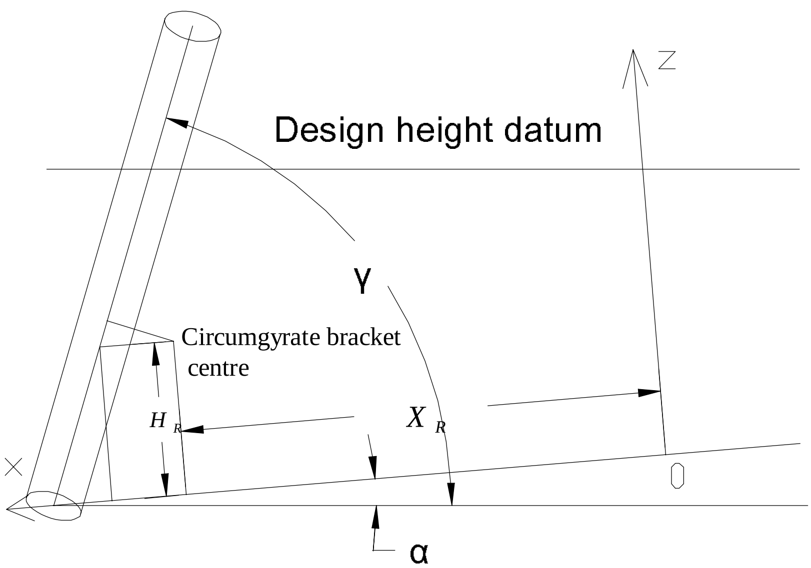

Fig. 5: 3D Coordinates of Picket Center in the Design Height Plane

The height of this intersetion point in the engineering coordinate system can be estimated by $h = h_0 - X \tan\alpha$, where h_0 is the height of the original point of SFCS. h_0 can be calculated according to the height of either GPS antenna in SFCS, the height measured by GPS, and the vertical and cross inclinations of the ship (Eq. 12).

$$h_0 = h_i - X_i\alpha - Y_i\beta - H_i \quad (12)$$

In practice, a mean h_0 can be obtained using the obeservations from three GPS receivers. The three–dimensional coordinates of the picket center in the design height plane can be estimated as:

$$X_D = X - \frac{h_D - h}{\sin\gamma}\cos(\gamma - \alpha),\ Y_D = 0,\ H_D = \frac{h_D - h}{\sin\gamma}\sin(\gamma - \alpha) \quad (13)$$

4.2 Picket Center Coordinate Computation Based on Precise Positioning Model

Under precise positioning mode, the distance between the picket and ship is not fixed. The coordinates of the picket center cannot be directly calculated. Two EDMs are used here to directly determine the coordinates of picket center in SFCS. As in Fig. 1, the position of the electronic distance instruments and the direction of the line of sight are determined in advance. The observed distances can be used to calculate coordinates of two points on the picket. When the picket is upright, the coordinates of the picket center can be deduced by intersecting line with a circle; whilst the picket is oblique, the coordinates of the picket can be estimated by intersecting a line with an ellipse. The short radius of the ellipse is the same as the picket circle radius r, and the long radius of the ellipse can be calculated using the inclination angle of the picket, i.e. $R = r * \sqrt{1 + 1/n^2}$, where n is the slope ratio of the picket. Suppose (X_0, Y_0) is the coordinates of the center of the ellipse, so the point on the ellipse shall satisfy the following equations:

$$\frac{(X_0 - X_R)^2}{R^2} + \frac{(Y_0 - Y_R)^2}{r^2} = 1 \quad (14)$$

$$\frac{(X_0 - X_L)^2}{R^2} + \frac{(Y_0 - Y_L)^2}{r^2} = 1 \quad (15)$$

Resolving the above combined equations the following results can be obtained:

$$Y_0(1) = \sqrt{-\frac{0.25*(X_R - X_L)^2 + R^2 {Y_L}^2 / r^2 - R^2}{k^2 + R^2 / r^2}} \quad (16)$$

$$X_0(1) = k * Y_0(1) + 0.5*(X_R + X_L) \quad (17)$$

or

$$Y_0(2) = -\sqrt{-\frac{0.25*(X_R - X_L)^2 + R^2Y_L{}^2/r^2 - R^2}{k^2 + R^2/r^2}} \quad (18)$$

$$X_0(2) = k * Y_0(2) + 0.5*(X_R + X_L), \quad (19)$$

where $k = \frac{X_R - X_L}{X_R - X_L} * (\frac{R}{r})^2$

The larger value of two resolutions is used as the picket centre coordinates.

4.3 Determination of the Height of the Picket Top

The infiltration and height of the picket top are two important control indexes in the pilling engineering. Infiltration is defined by the changes of the height of the picket top during a number of hammer beats. A vidicon is used to read the reading on the picket, marked as a red point by a EDM, and then the height of the picket top is reckoned. The calculation can be conductde in two steps: (1) the height reading on the picket; (2) estimation of the height of the picket top.

(1) Determination of the height of a EDM:

$$h_i = h_0 + X_i\alpha + Y_i\beta + H_i \quad (20)$$

(2) The height reading on the picket

$$Hh = Hc + (Xh - Xc)*\alpha + (Yh - Yc)*\beta \quad (21)$$

where ($Xh \ \ Yh$) is the coordinates of the red point blazed by a EDM, and ($Xc \ \ Yc$) is the coordinates of EDM.

(3). Determination of the height of picket top

The height of the picket top Hd can be estimated using the picket gradient. The reading on the picket is from the bottom to the top. The length L from the reading to the top of the picket equals to the length of the picket subtracted by the reading.

The height of the picket top (upright picket): $Hd = Hc + L$

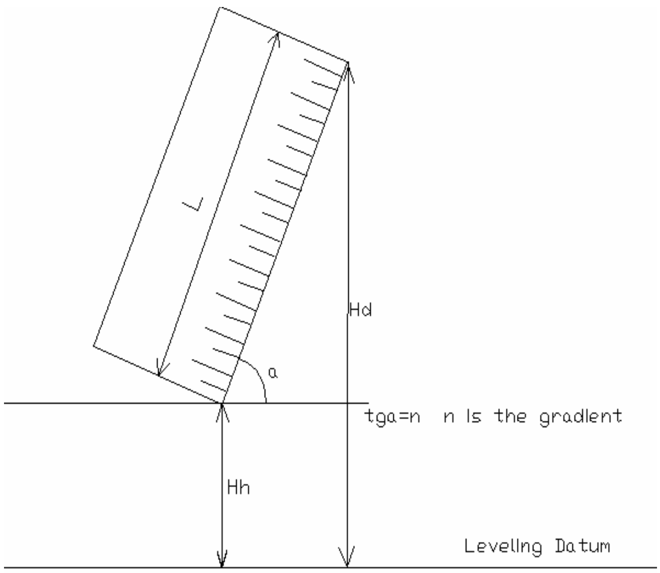

Fig. 6: Height of the Picket Top (inclined picket) (a)

The height of the picket top (inclined picket) as in Fig. 6:

$$Hd = Hc + L * \sin(arctg(n)) \quad (22)$$

The height of the picket top (inclined picket) as in Fig. 7:

$$Hd = Hc + L * \sin(arctg(n)) - 2R * tga \quad (23)$$

5. MEASUREMENTS AND COORDINATE DETERMINATION

Instrument installation is an important part to ensure the stabilization of whole positioning system. The GPS receiver antennas shall be mounted at locations with open view to aviod the signal obstruction caused by the pickets and ship. The coordinates of GPS1, GPS2 and GPS3 must be determined after instrument installation while the ship is relatively level. Some control points are set up on the ship deck for this purpose and also used for future check. The vertical automation compensation device shall be switched off before starting the measurements. Angles, distances, and the height differences are measured to obtain 3D coordinates using least sqaures adjustment. The coordinates of the picket center shall also be determined along with the coordinates of the EDMs and the circumyrate center. Reflecting stick slices are used for the places where using prisms are inconvenient.

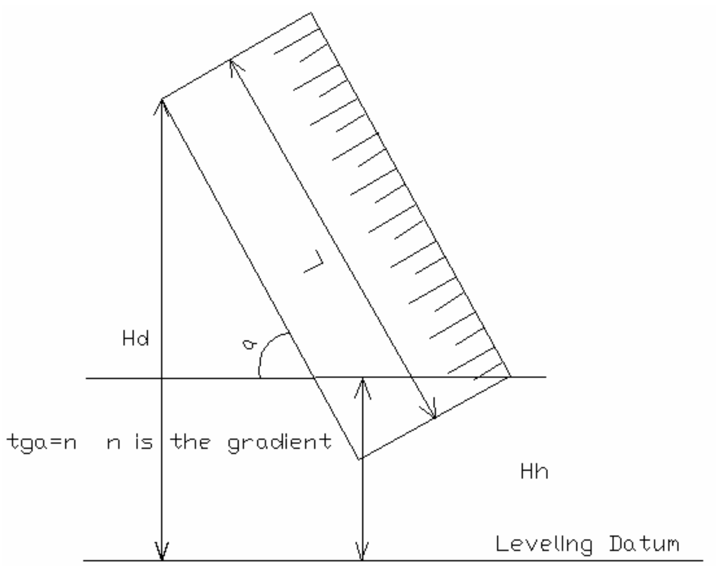

Figure 7: Height of the picket top (incline picket) (b)

Indirect method is adopted to determine the coordinates of the picket center. Two points on the same plane are determined with the reflecting stick slices. When the radius of the picket is known, the coordinates of the picket center can be calculated. Coordinates of a point and an azimuth are assumed as the unknowns for the adjustment computation to obtain the coordinates of the GPS1, GPS2, GPS3 and the picket center. Then the SFCS can be defined to determine other coordinates.

Coordinates shall be calculated for all unknown points before the adjustment computation. The approaches can be used including length intersection, angle intersection and traverse adjustment calcualtion. The error equation is expressed as:

$$V = A\delta X + L \quad (24)$$

6. CASE STUDY

Fig. 8 is the piling ship using in the trial, "Haizhuang 8". The picket height of the ship is 92 m, and it can raise piles up to 200 tons. Three GPS receivers and two EDMs are installed on the ship deck.

In order to determine the coordinates of the entire system, S1, S2, S3 are the control points set on the ship deck as shown in Fig. 9. R1 and L1 are the reflecting stick slice points on the picket, and T is the picket center. D1 and D2 are the centers of the EDMs. The distances and angles in Fig.9 are measured using a total station. Suppose the S2 be the original point and the direction from S2 to S1 be the positive direction of Y axis. The coordinates for S2 and S1 are (0, 0) and 0, 48.960

All the coordinates of the points can be obtained after the adjustment and listed in Table 1. The calculated length of OT is 50.961 m, and the center of connecting line from GPS1 to GPS2 has the coordinate as (1.2466 , 1.3857). The center is the original point of the SFCS. Based on above two points, the whole coordinates can be transformed to the ship-fixed coordinate system, and new coordinates of the points are listed in Table 2. The transformation parameters of two coordinate systems are listed in Table 3.

Fig. 8: Hahizhuang 8

Triangle height measurement is used by total station to obtain the height differences of each two points as in Fig. 10. Great effort is paid to set two electronic distance instruments at the same height.

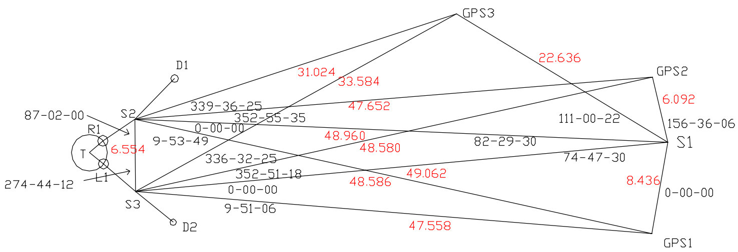

Fig 9: Observationsof the horizontal control network

Table 1: Coordinates under the presumed coordinate system

ID	X	Y
S1	.0000	.0000
S2	.0000	48.9600
S3	6.5632	48.6212
GPS3	-10.8054	19.8823
GPS1	8.3584	1.1025
GPS2	-5.8652	1.6688
D1	-1.2260	47.2012
R1	2.2457	51.1317
D2	7.5576	46.7989
L1	4.4810	51.0445
T	3.4106	52.3002

Table 2: Coordinate under the ship-fixed coordinate system

ID	X	Y
T	50.961	.000
S1	-1.437	1.187
S2	47.478	3.266
S3	47.419	-3.306
GPS3	17.968	12.827
GPS2	.019	-7.117
GPS1	-.019	7.117
D1	45.669	4.416
R1	49.744	1.114
D2	45.640	-4.377
L1	49.751	-1.123

Table 3: Transformation parameter

Translation in x	-1.4374(m)
Translation in y	1.1866(m)
Scale factor	.042464316(ppm)
Rotation	-206078.8854(")

The height observations are listed in Table 10. Adjustment computation is conducted after the closure verification. The results are listed in Table 4.

Table 4: Height of the instrument

ID	Height(m)	MSE(m)
S1	4.450	0.015
S2	-1.117	0.002
S3	-1.108	0.002
GPS1	11.055	0.016
GPS2	11.002	0.016
GPS3	10.991	0.016

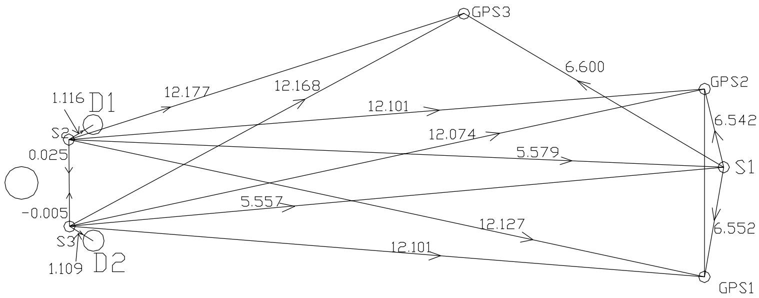

Fig.10: Height measurement network

The test piling was conducted after hardware and software installation. The position result is consistent compared with traditional method but with much higher efficiency. The piling result can be seen from Fig. 11.

Fig. 11: Result of the piling with "Haizhuang 8" piling ship

7. CONCLUSION

Limitations of the position of the piling can be found with tranditional surveying technology because of the observation range, real time compuation capability and other instrument restraints. The adoption of real-time kinematic Global Positioning System (RTK-GPS) can integrate the field measurements with the real-time calculation into one positioning system, which is specially useful in the sea piling engineering. Three GPS instruments are used for the kinematic positioning in the proposed system in order to calculate the positions of the picket, whilst two electronic distance measurement instruments are used to achieve an even higher accuracy. Meantime, a 2D tiltmeter is used to measure the altitude of the ship. Data communication between the control centre and the instruments is considered. Actually, for the ease in application, both the parameter computation and optimised data and graphic interfaces for the ship controlling are realized by the software.

The RTK-GPS system is applied for piling engineering in the longest cross sea bridge, Donghai Bridge in Shanghai, China. When applying the proposed piling system for the biggest steel picket in Asia, the position time can be controlled within half an hour, and its position accuracy is around 5 cm, which can satisfy the request of the engineering completely.

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BIOGRAPHICAL NOTES

Dr. Chun Liu works as an associate professor at the Department of Surveying and Geo-Informatics, Tongji University, Shanghai, P. R. China. He got his PhD in Geographical Information System (GIS) three years ago from Tongji University, and is currently working in spatial data quality control and system integration of 3S technologies. His research interests focus on spatial data modelling, spatial data quality, image data processing, and integrated geographical data and GPS measurements for engineering applications. As a research assistant at Hong Kong Polytechnic University from 2001 to 2003, he worked for a project, titled "Development of Infrastructure for Cyber Hong Kong". So far, he has published about 40 papers in Chinese and in English. In recent years, he gave several oral presentations in a variety of international symposia.

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