Variance-based Sensitivity Analysis for Model Evaluation in Engineering Surveys

Volker SCHWIEGER, Germany

Key words: variance-based sensitivity analysis, FOURIER amplitude sensitivity test, vehicle positioning, kinematic KALMAN filter, GPS, DGPS, dead reckoning.

SUMMARY

A general sensitivity concept, namely the variance-based sensitivity analysis suited for all kind of models, is presented. The variance-based analysis is sampling-based and therefore applies Monte Carlo simulation. Moreover, it relies on the computation of conditional variances. Measures that do not need a linear or additive model behavior for quantitative sensitivity analysis are SOBOL's and the FOURIER amplitude sensitivity test (FAST) sensitivity indices. The main advantage of the methods is that the analytic structure of the model to be analyzed has not to be known. Theoretically an unknown computer code may be used as model for the sensitivity analysis.
The paper will deal with a kinematic KALMAN-filter used for vehicle positioning as application for the variance-based method. The state vector used for the filter algorithm consists of the coordinates of the vehicle, its velocity and its orientation. The sensitivity analysis identifies the influence of the measured quantities like GPS coordinates, orientation changes and distance increments on the state vector components.

ZUSAMMENFASSUNG

Ein übergreifendes Konzept zur Sensitivitätsanalyse, die varianz-basierte Sensitivitätsanalyse, wird vorgestellt. Da zur varianz-basierten Analyse Stichproben generiert werden, ist sie grundsätzlich als Monte Carlo Simulation zu betrachten. Die Sensitivitätsmaße bauen auf der Berechnung bedingter Varianzen auf. Die Maße nach SOBOL oder die Indizes des FOURIER Amplituden Sensitivitätstests erfordern kein lineares oder additives Modellverhalten um zu quantitativen Sensitivitätsaussagen zu kommen. Der Hauptvorteil der Methode liegt folglich in der Tatsache, dass die analytische Struktur des zu untersuchenden Modells nicht bekannt sein muss. Im Extremfall kann ein unbekannter Computer Code analysiert werden.
Der Beitrag beschäftigt sich mit einem kinematischen KALMAN-Filter zur Fahrzeugortung als Anwendung der varianz-basierten Methode. Der Zustandsvektor, der für den Filteralgorithmus benötigt wird, enthält die Koordinaten, die Geschwindigkeit und die Orientierung des Fahrzeuges. Die Sensitivitätsanalyse identifiziert den Einfluss der gemessenen Größen wie die GPS Koordinaten, die Richtungsänderungen und die Streckeninkremente auf die Zustandsgrößen.

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1. INTRODUCTION

If an engineering surveyor talks about sensitivity analysis, he addresses the sensitivity of a monitoring network with respect to expected movements of the object to be monitored (e.g. NIEMEIER 1985). The sensitivity of a monitoring network is defined by the possibility to detect the expected movements integrated in a deformation model. This definition is a short-come with respect to more general sensitivity concepts of other disciplines like e.g. economy or risk assessment.

Within this paper a general sensitivity concept, namely the variance-based sensitivity analysis suited for all kind of models, is presented. The variance-based analysis focuses on the questions “which of the input variables variances influences the model output variance at most?” and “which of the input variables has to be known more accurate to reduce the output variance?”. By answering these questions the application fields of sensitivity analysis to validate and optimize models in engineering surveys is widely extended.

Furthermore the application of sensitivity analysis dealing with expected deformations is restricted to linear or linearized models. This deficiency will be overcome due to the model-independence of the quantitative results of variance-based sensitivity analysis.

2. BASICS OF VARIANCE-BASED SENSITIVITY ANALYSIS

2.1 Basics of Sensitivity Analysis

According to SALTELLI et al. (2000) sensitivity analysis is the study about the relations between the input and the output of a model. Originally sensitivity analysis has dealt, like system theory, with the variation respectively the uncertainties of the input quantities. Later on the item input has been extended to uncertainties of model parameters and the general model structure.

The variation of the input leads to changed output quantities. The relation between varied input and output is measured by different sensitivity measures that are the basis for model validation and optimization. Figure 1 gives an overview on the general procedure of sensitivity analysis.

Sensitivity analysis methods may be categorized according to the outcome of the related sensitivity measures: qualitative or quantitative methods, local or global methods and methods that depend on or are independent of the model characteristics. SALTELLI et al. (2000) give an overview and further insight into the different categorizations. The following chapters focus on variance-based methods, because they deliver global, quantitative and model-independent sensitivity measures. The main point will be the application of the FOURIER amplitude sensitivity test (FAST) as one method of variance-based sensitivity analysis.

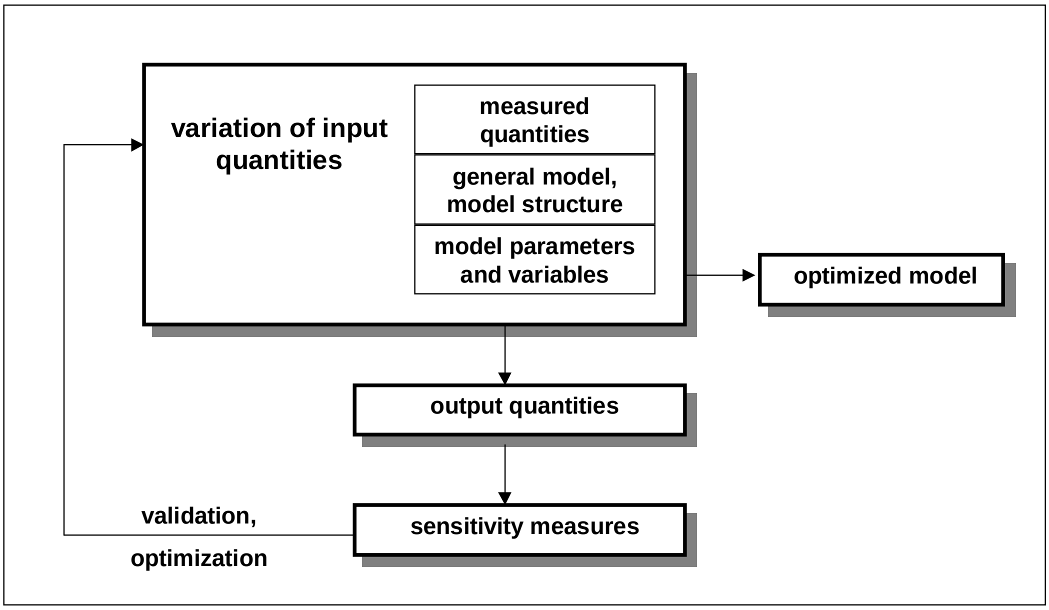

Figure 1: General procedure for sensitivity analysis

2.2 Variance-based Sensitivity Analysis

Variance-based sensitivity analysis is based on generated samples. Due to this fact it is understood as a subset of the sample-based methods respectively Monte Carlo based methods. Further Monte Carlo based sensitivity analysis methods are regression and correlation analysis as well as analysis of rank transformed data. Their deficiency is dependence of the quantitative results on the model characteristics like linearity or additivity.

The general procedure to get sensitivity measures for sample-based sensitivity analysis methods is given in the following:

- Definition of probability distributions functions for the input quantities
- Generation of samples from the defined probability distributions
- Evaluation of the model using the generated sample
- Analysis of the output variance
- Sensitivity analysis of the output variance in relation to the variation of the input quantities

In variance-based sensitivity analysis it is assumed that the true value $\tilde{X}_i$ of an input quantity X_i is known. A conditional variance $\sigma^2_{E(Y/\tilde{X}_i)}$ is estimated holding the true value of the respective input parameter fixed. Unfortunately, in general the true values of the input parameters are not known. Therefore the expectation value $E(Y/X_i)$ above the whole variation interval of the input quantity X_i has to be evaluated to get a global sensitivity measure. This conditional variance $\sigma^2_{E(Y/X_i)}$ is an essential part of all variance-based sensitivity measures.

Variance based sensitivity indices are estimated as ratios between the conditional variance and the unconditional variance σ_Y^2 for the output quantity Y

$$S_i = \frac{\sigma^2_{E(Y/X_i)}}{\sigma_Y^2}. \tag{1}$$

These ratios are called correlation ratios, importance measures or first order sensitivity indices. They provide quantitative information about the importance of the input quantities. Assuming non-correlated input,

$$\text{for additive models } \sum_{i=1}^{n} S_i = 1 \tag{2}$$

holds true. This leads to an easy quantitative interpretation of the sensitivity indices, because each S_i delivers a direct measure for the portion of X_i on the output variance σ_Y^2. For non-additive models the interactions among the input quantities within the model have to taken into account; meaning the consideration off effects of higher order. Non-additive models need a complete decomposition of the function Y into summands of increasing order involving a more sophisticated form for equation (2)

$$\sum_{i=1}^{n} S_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} S_{i,j} + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n} S_{i,j,k} + \ldots + \sum \cdots S_{i,j,k,\ldots,n} = 1 . \tag{3}$$

Equation (3) is valid for non-correlated input only. The terms of higher order are estimated by holding more than one input quantity fixed; e.g. the two input quantities X_i and X_j

$$S_{i,j} = \frac{\sigma^2_{E(Y/X_i,X_j)}}{\sigma_Y^2} - S_i - S_j . \tag{4}$$

The computation of all higher order terms is connected with high computational costs, especially for complex models. This leads to the estimation of total effects S_{Ti} with respect to an input quantity X_i. They include all higher order terms with respect to the input quantity X_i. The total effect is computed as follows

$$S_{Ti} = S_i + \sum_{j=1}^{n} S_{i,j} + \sum_{j=1}^{n}\sum_{k=j+1}^{n} S_{i,j,k} + \ldots\ldots + \sum \cdots S_{i,j,k,\ldots,n} \quad \text{mit } j,k \neq i,\ j \neq k. \tag{5}$$

The estimation of the total effect may be realized by the help of the conditional variance $\sigma^2_{E(Y/X_1,X_2,\ldots X_{i-1},X_{i+1},\ldots,X_n)}$, for which all input quantities apart from X_i have to be held fixed. This conditional variance may be abbreviated in the form $\sigma^2_{E(Y/X_{\sim i})}$, the respective total effect is computed according to

$$S_{Ti} = 1 - \frac{\sigma^2_{E(Y/X_{\sim i})}}{\sigma_y^2}, \tag{6}$$

meaning that we estimate the total effect with one computational step like the first order effects (see equation (1)). The first order sensitivity indices are a quantitave sensitivity measure for additive models and the total effects are a quantitaive measure for all kind of models independent of their model characteristics (SALTELLI et al. 2000). Equation (2) may be used to validate the additivity of a model. Equivalently a comparison between S_i and S_{Ti} may lead to a conclusion regarding the additivity of models with non-correlated input

$$
\begin{aligned}
S_{Ti} &= S_i \quad \text{additive model,} \\
S_{Ti} &> S_i \quad \text{non-additive model.}
\end{aligned} \tag{7}
$$

2.3 FOURIER Amplitude Sensitivity Test (FAST)

For the estimation of the conditional variances $\sigma^2_{E(Y/X_i)}$ and $\sigma^2_{E(Y/X_{\sim i})}$ different methods are in use. Within the context of this paper we will deal with the sensitivity indices that are estimated in the frequency domain. The method is named FOURIER amplitude sensitivity test (FAST), because a FOURIER series expansion is required for the transformation into the frequency domain. The method was developped by CUKIER et al. (1973), CUKIER et al. (1975) and SCHAIBLY / SHULER (1973) and has the advantage of a small sample size in comparison to the method of SOBOL (SOBOL 1993).

The FAST transforms the multidimensional integral given as sum in equation (3) into a one-dimensional one

$$E(Y) = \int \ldots \int Y(\boldsymbol{X}) \cdot p(\boldsymbol{X})\, dx_1 \ldots dx_n = \int f(s) ds\,, \tag{8}$$

with $p(X)$ as multidimensional probability function and

$$f(s) = f(x_1(s), x_2(s), \ldots, x_n(s))\,, \tag{9}$$

as a function that depend on one parameter s only. Each input quantity X_i is assigned to one frequency ϖ_i by the general equation

$$x_i(s) = g\left(\sin(\varpi_i \cdot s)\right) \quad \text{für} \quad i = 1,2,\ldots,n\,. \tag{10}$$

According to SALTELLI at al. (1999) the function

$$x_i(s) = \frac{1}{2} + \frac{1}{\pi} \cdot \varpi_i \cdot s\,, \tag{11}$$

is the best one to cover the one-dimensional space uniformly. It is used for the simulations in this paper. The chosen frequencies ϖ_i have to be independent of each other. To avoid alias effects the higher harmonics up to order four or six have to be independent of each other too (SCHAIBLY / SHULER 1973). The function $f(s)$ may be described by FOURIER series. The associate FOURIER coefficients are used to estimate the unconditional s_Y^2 and the conditional variances s^2_{Y/X_i}

$$
\begin{aligned}
s_Y^2 &= \frac{1}{2} \cdot \sum_{k=1}^{\infty} (A_k^2 + B_k^2), \\
s^2_{Y/X_i} &= \frac{1}{2} \cdot \sum_{p=1}^{m} (A^2_{p\varpi_i} + B^2_{p\varpi_i}),
\end{aligned} \tag{12}
$$

with A and B as the FOURIER cosine respectively sine amplitudes and m as order of the higher harmonics that are considerd. These variances are used to compute estimates for the first order sensitivity indices S_i^{FAST} as given in equation (1)

$$S_i^{FAST} = \frac{s^2_{Y/X_i}}{s_Y^2} \tag{13}.$$

Extended FAST deals with the estimation of total effects S_{Ti}^{FAST}. To estimate them we may use

$$s_{Y/X_{\sim i}}^2 = \frac{1}{2} \cdot \sum_{p=1}^{m} (A_{p\varpi_{\sim i}}^2 + B_{p\varpi_{\sim i}}^2) \text{ and } S_{Ti}^{FAST} = \frac{s_{Y/X\sim_i}^2}{s_Y^2}, \tag{14}$$

with A and B as the FOURIER coefficients not in connected to input quantity X_i respectively the assigned frequency ϖ_i (SALTELLI / BOLADO 1998). The defintion and the assignment of the frequencies $\varpi_{\sim i}$ is described in detail by SALTELLI et al. (1999). A complete and detailed description of variance-based sensitivity analysis is beyond the scope of the paper. The autor refers to SALTELLI et al. (2000) for a general overview and to SCHWIEGER (2004) for an overview in the context of geodetic applications.

3. APPLICATION TO VEHICLE POSITIONING

3.1 Data Acquisition and Evaluation for Vehicle Positioning

One research focus at the institute for applications of geodesy to engineering is the kinematic positioning of vehicles. For this reason a kinematic KALMAN filter was developed that is capable to integrate different sensors in a common evaluation model. For the application presented in this article a multi-sensor system consisting of the sensors given in table 1 is used. DGPS delivers absolute coordinates Y and X. Additionally dead reckoning sensors are used: the differential odometer and the gyroscope measure orientation changes $\Delta\alpha$ and the differential odometer and the non-contact optical speed and distance sensor measure distances Δs.

Table 1: Sensors for vehicle positioning (RAMM / SCHWIEGER 2004)

Sensor	measured quantities	resolution	accuracy
DGPS	Y, X	< 1 m	1 – 3 m
differential odometer	Δs, $\Delta\alpha$	2 mm, 0,1 gon	0,4 %
speed and distance sensor	Δs	1,9 mm	0,1 %
gyroscope	$\Delta\alpha$	0,2°/s	0,3°/s

The state vector of the KALMAN filter contains the coordinates of the vehicle, its velocity and its orientation. These state vector quantities are estimated every 0,2 seconds in accordance with the data acquisition rate. More details regarding the sensors and data acquisition are given in RAMM / SCHWIEGER (2004) The used trajectory related KALMAN filter is proposed firstly by AUSSEMS (1999).

3.2 Variance-based Sensitivity Analysis for Vehicle Positioning

For the Monte Carlo simulation the standard deviations according to the accuracy column of table 1 are used assuming normal distribution of the measures quantities. A sample size of 9750 is chosen to get reliable results and the extended FAST method was used due to the non-linear and non-additive equations of the KALMAN filter (compare SCHWIEGER 2004).

The sensitivity analysis was carried through for 6 filter epochs and different driving characteristics. The 6 input quantities measured by the same sensor at different epochs quantities are grouped; thus one group of 6 input quantities is regarded as one input leading to 6 inputs. The driving characteristics are divided in curved and non-curved variants (compare table 2).

Table 2: Overview about driving variants; left non-curved, right curved

abbreviation	velocity / acceleration	abbreviation	radius $[m]$	velocity $[m/s]$
A1	5,6 m / s	K1	10	5,6
A2	13,9 m / s	K2	22	13,9
A3	27,8 m / s	M1	100	5,6
B1	5 m / s^2	M2	100	13,9
B2	10 m / s^2	M3	100	17,6
B3	25 m / s^2	M4	100	27,8
C1	- 5 m / s^2	G1	1000	5,6
C2	- 10 m / s^2	G2	1000	13,9
C3	- 25 m / s^2	G3	1000	17,6
D1	25 m / s^2 up to ep3	G4	1000	27,8
D2	- 25 m / s^2 up to ep3			

The following figures 2 and 3 show examplarily the results of the variance-based sensitivity analysis for the state quantity orientation α in dependence of the measured quantities Y-DGPS-coordinates, X-DGPS-coordinates, orientation changes by differential odometer (da-odo) and by gyroscope (da-gyr) as well as distances by differental odometer (ds-odo) and by non-contact optical speed and distance sensor (ds-spe). The figures present the total effects as computed according to equation (14).

Due to the fact that the sum of the total effects are clearly higher than one the model is non-additive with respect to α. The influence of the measured DGPS coordinates is by far the largest on the orientation α .They account for approximately 70 % of the total variance for all variants. Figure 2 additionally shows that an increase of velocity leads to an increase of non-additivity and to an increase of influence of the non-DGPS input quantities. The radius has no influence on the variance-based sensitivity measures. This may be seen in figure 3; e.g. the drives K1, M1 and G1 are simulated using different radii but the same velocity. The sensitivity measures remains unchanged. For further investigations regarding vehicle positioning and other application in egineering survey is referred to SCHWIEGER (2004).

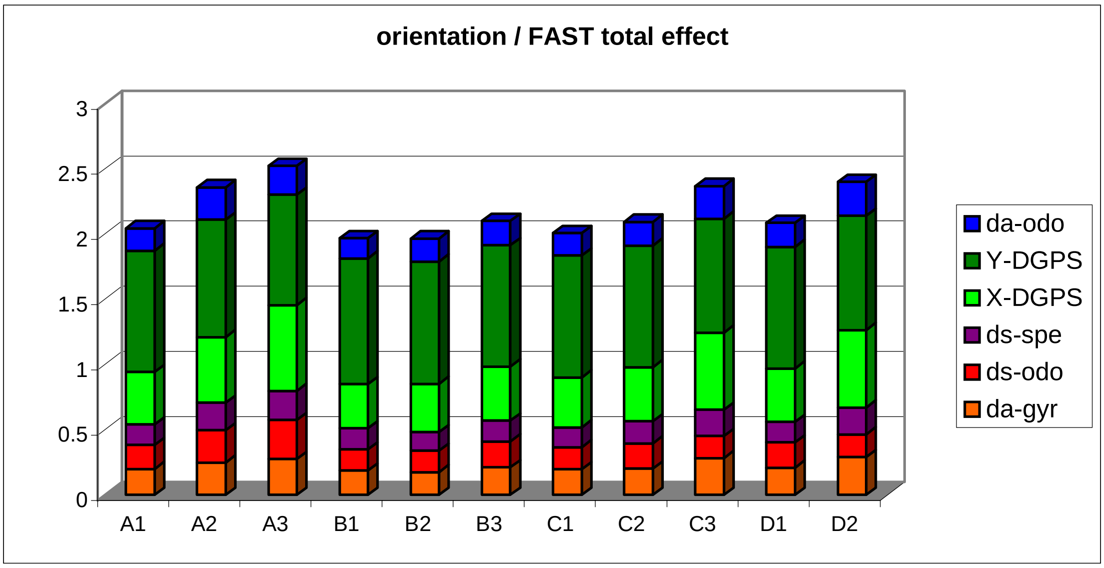

Figure 2: Total effects for orientation for non-curved drives

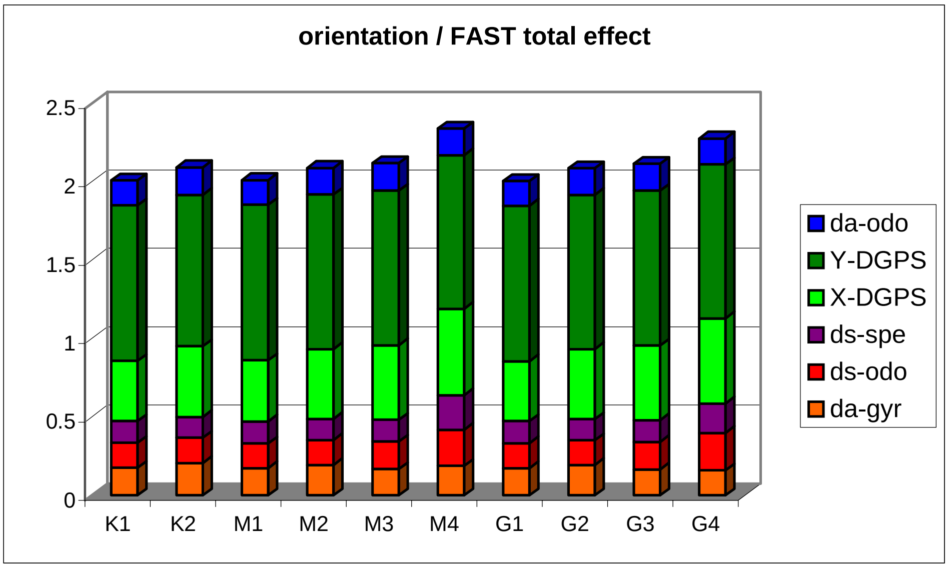

Figure 3: Total effects for orientation for curved drives

The results of the sensitivity analysis show that the measured DGPS coordinates have the strongest influence on the state vector meaning that an increase in DGPS-acquisition accuracy leads to a strong increase in state vector accuracy. An increase in acquisition accuracy of the other measured quantities will have less effect.

4. SUMMARY AND OUTLOOK

The variance-based sensitivity analysis was used to identify the portions of the variance related to different measured input quantities. This method for sensitivity analysis is independent of the characteristics of the analyzed model. The results may be used for future model optimization; e.g. by comparing different kinematic approaches or model parameters. The first prove for beneficing was given within this paper. The author proposes to use the method for other models and algorithms and to exchange experiences.

Finally it has to be repeated that the equations presented are valid for non-correlated input only. Quantities measured for geodetic purpose are of correlated nature frequently. Therefore investigations regarding sensitivity measures for correlated input will be a future task.

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BIOGRAPHICAL NOTES

1964	born in Springe, Germany
1983 – 1989	Study of geodesy in Hannover
1989	Dipl.-Ing. in geodesy (Hannover)
1991 – 2000	Scientific associate at Geodetic Institute, University of Hannover
1998	Dr.-Ing. in geodesy (Hannover)
2000 – 2001	Scientific associate at GeoForschungsZentrum Potsdam
2002 – 2003	Scientific associate at Institute for Applications of Geodesy to Engineering, University of Stuttgart
2003 - now	Head of department “Positioning and Measurement Techniques” at Institute for Applications of Geodesy to Engineering, University of Stuttgart
2004	Dr.-Ing. habil. (habilitation / Stuttgart)

CONTACTS

Volker Schwieger
Institute for Applications of Geodesy to Engineering
University Stuttgart
Geschwister-Scholl-Str. 24D
70174 Stuttgart
GERMANY
Tel. + 49 711 1214 064
Fax + 49 711 1214 044
Email: volker.schwieger@iagb.uni-stuttgart.de
Web site: http://www.uni-stuttgart.de/iagb/